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THESIS

CHAOS THEORY AND INTERNATIONAL RELATIONS

by

Dimitrios Kantemnidis

December 2016

Thesis Advisor: James Luscombe
Co-Advisor: Florina Cristiana Matei

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Employing a theory from the natural sciences to analyze a topic of social sciences is a procedure that can benefit decision makers, who can avoid mistakes by testing their decisions with the help of mathematical models. This thesis provides an overview of Chaos Theory—why it has been accepted in the natural sciences, specifically in physics—and whether it can be relevant for the IR domain of social sciences. The applicability of Chaos Theory to the physics domain is examined through the OGY (Ott, Grebogi, Yoke) method and its applications. For the international relations domain, Chaos Theory is modeled in two specific international relations puzzles, bipolarity and democratic peace, to show the utility of the theory in this social science field. The results of the model are compared with the conventional international theories of Liberalism and Realism. The comparative analysis between the use of Chaos Theory in physics and in international relations issues, respectively, shows that for the former we have controllability of chaotic phenomena, and for the latter, it is applicable and helpful. This thesis concludes that the theory of Chaos is a universal theory that is applicable to both natural and political sciences.

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CHAOS THEORY AND INTERNATIONAL RELATIONS

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Submitted in partial fulfillment of the
requirements for the degree of

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I. INTRODUCTION

A. MAJOR RESEARCH QUESTION

The famous Chaos Theory (CT) is a theory of physics that promises to help us predict the unpredictable. The wild, perplexing, and unpredictable behavior of a physical system with sensitivity to its initial conditions was named chaotic behavior by physicists.[1]

During the 20th century, Henri Poincare, Yoshisuke Ueda, and Edward Lorenz were the pioneers of the study of CT, although they never used the term Chaos. They studied the behavior of complex and unpredictable physical systems and they found that this behavior was not random.[2] It took around a century for their work to become widely known, but in the last four decades—with the development of computer science and the ability to analyze huge amounts of data—CT has been studied extensively by natural scientists, and interest in the theory has expanded to other physical sciences like chemistry, biology, and electronics.

Until Poincare's work, physicists used Newtonian classical physics, which was not applicable on several experiments (that contained chaotic phenomena), because of its deterministic nature.[3] Sufficiently complicated systems, like a glass of water, could not be explained with the Newtonian paradigm as there are few experimentally measurable quantities, and it was assumed that whatever could not be modeled was noise.[4] For that reason, the mathematical models most commonly encountered in physics had the property of being linear.[5] Nonlinear equations were difficult to handle, and our

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[1] Edward Ott and Mark Spano, "Controlling Chaos," *AIP Conference Proceedings* 375, no. 92 (1996): 92.

[2] I explain the difference between random and non-random phenomena, as well as the distinction between Chaos and chaos, in Chapter I, section C.2, "Randomness, Chaos, and chaos."

[3] Deterministic for physicists is the situation in which known initial data leads to predictable outcomes.

[4] It is common for physicists to treat as noise (random effects) any effects of a system that are not modeled.

[5] I explain the difference between linear and nonlinear in Chapter I, section C.1, first paragraph.

knowledge of them was poor; that is, until the work of Poincare. What the physicists learned with CT is that seemingly random behavior can emerge from deterministic models.

While for physics experiments the applicability of CT is clearly indicated, little is known about CT for domains that are not part of the natural sciences; however, that has changed recently. International Relations (IR) theorists have also taken a keen interest in the science of CT. Just as physicists named the dynamic behavior of a physical system "chaotic," political scientists, too, have characterized as "chaotic" the unpredictable behavior of the international relations system (and also dependent upon initial conditions[6]).[7] In the realm of political sciences, and specifically for the international relations field, CT appears to be a new and promising tool. Dylan Kissane argues that "the assumption of chaos can assist in explaining the variety of international behavior exhibited by international actors, and also the recurring behaviors that have been previously explained away by references to anarchy and its implications for the wider system."[8] In the same vein Alvin Saperstein stresses the importance of "the physicist's mode of thinking to the modeling of international relations."[9] He explains that it is feasible "to develop the idea of 'chaos' in a deterministic international system, and to apply it to simple mathematical models of the interactions between competing states in such a system."[10]

Under these circumstances, this thesis answers one primary question and two secondary questions:

---

[6] With the term "initial conditions" the natural scientists mean: the conditions at an initial time $t = t_o$ from which a given set of mathematical equations or physical system evolves.

Eric W. Weisstein, "Initial Conditions." From MathWorld—A Wolfram Web Resource. http://mathworld.wolfram.com/InitialConditions.html.

[7] Dylan Kissane, "A Chaotic Theory of International Relations?," *Pro Polis,* no. 2 (2007): 91–92.

[8] Kissane, "A Chaotic Theory of International Relations?" 91.

[9] Alvin M. Saperstein, *Dynamical Modeling of the Onset of War* (Singapore: World Scientific Publishing, 1999), 3.

[10] Ibid., v.

-Is the Chaos Theory a universal theory, with clear application for both physics and international relations fields?

-Can CT be utilized in physics to achieve control of chaotic phenomena in physical systems?

-Can CT be utilized in the IR field, to explain complex phenomena by modeling (with chaotic models) the real world?

B. IMPORTANCE OF THE STUDY

Certain aspects of human nature can be explained by classical Newtonian mechanics as long as these phenomena fall within a predictable, quantifiable range. A theory that promises to explain nonlinear phenomena that appear to be random or unexpected will be the new lenses that scientists need for examining the complex problems of our age. The motion of a double pendulum and the Arab Spring represent such complex phenomena for physicists and the IR analysts, respectively.

The importance of such research for the IR field is that if CT works both for physics and the international relations domain, IR theorists will be able to predict better the future relations between states, war prone situations, and the possible results of a state's action. If Chaos Theory helps physicists to predict the future behavior of a physical system, it is also possible to help political scientists to predict future international relations' trends, which in turn will help policymakers in their decisions.

C. LITERATURE REVIEW

A bit of dialog from the novel Jurassic Park is: "They believed that prediction was just a function of keeping track of things. If you knew enough, you could predict anything. That's been cherished scientific belief since Newton. Chaos Theory throws it right out the window."[11] Michael Crichton, the famous novelist, expresses the

[11] Michael Crichton, goodreads.com, accessed December 14, 2016, http://www.goodreads.com/quotes/tag/chaos-theory.

revolutionary nature of Chaos Theory that made a lot of scientists, other than physicists, to try combining CT with the conventional theories of their science. There is a rich literature on CT as far as the natural sciences are concerned, and in the last decade there has been a growing interest in the IR domain. In this section, I describe the evolution of CT, the confusion that the term Chaos causes in literature, and the use of CT by IR theorists.

1. Chaos Theory and the Natural Sciences

The first body of literature discusses the evolution of CT through the efforts of physicists to handle nonlinear problems. According to Professor James Glenn, CT examines systems that are characterized by "erratic fluctuations, sensitivity to disturbances, and long term unpredictability."[12] For a system to exhibit chaos, its equations of motion must be nonlinear (but nonlinearity does not guarantee Chaos).[13] To understand the difference between linear and nonlinear, we can say that almost all of the linear equations of mechanics are analytically solvable but almost none of the nonlinear ones are.[14] James Gleick maintains that linear systems are such that you can take them apart, and put them together again; the pieces add up. Nonlinear systems generally cannot be solved and cannot be added together.[15]

It is very common for mathematicians and physicists in their textbooks to focus on linear problems, and when they have to handle a nonlinear problem, they often solve the problem using approximations that reduce it to a linear problem. The first person to notice some of the symptoms of Chaos was the French mathematician Henri Poincare during an effort to solve the gravitational three-body problem. In 1887, King Oscar II

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[12] James E. Glenn, *Chaos Theory: The Essentials for Military Applications* (Newport, RI: Naval War College Press, 1996), xiv.

[13] John R. Taylor, *Classical Mechanics* (Colorado: University Science Books, 2005), 458–459.

[14] Ibid., the linear equations obey the rule: $f\left(x+a\right)=f(x)+f(a), f(ax)=af(x)$.

[15] James Gleick, *Chaos Making a New Science*, 20th anniversary ed. (New York: Viking Penguin Inc., 1987), 23.

offered a prize for the scientist who could solve that unsolved problem.[16] Poincare's version of the solution contributed some ideas that would lead to the theory of chaos; he won the prize.[17]

Poincare's paper was published around January 1890 and until the end of 1950s no progress on CT was made.[18] As Paul R. Gross and Norman Levitt maintain, there are three main reasons that can explain the delay and prove that sometimes timing is the most important factor to achieve a goal.[19] First and foremost, the astounding scientific developments of special and general relativity and quantum mechanics absorbed the lion's share of intellectual energy during these years. Second, the theory of Chaos depends on fundamental mathematics such as topology, differential equations, and computational complexity, which was developed long after Poincare's days. Finally, high-speed electronic computers with developed processors were essential for CT to grow, as there are complex shapes and pictures—as a result of the complex trajectories of the chaotic equations—on which the scientists must rely to inform their concepts.

As Ralph Abraham and Yoshisuke Ueda state in their book, after Poincare's work, CT grew along parallel lines.[20] From 1961, Ueda in Kyoto worked on chaotic attractors. During the same years Edward Lorenz in Cambridge worked on what became known as the "Lorenz attractor," and Christian Mira in Toulouse worked also on complex dynamical phenomena[21] The physicist Abraham states that "after a meeting at the New York Academy of Sciences in 1979, which brought many of the Chaos pioneers together

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[16] The three-body problem is to solve the equations that describe the motion of three bodies that interact according to the laws of Newtonian mechanics.

[17] His solution combined the unstable periodic motion with the complicated dynamical behavior.

[18] Ralph Abraham, "The Peregrinations of Poincare," Mathematics Department, University of California, Santa Cruz, 2012, accessed September 7, 2015, http://www.ralphabraham.org/articles/MS%23136.Poincare/ms136.pdf.

[19] Paul R. Gross and Norman Levitt, *Higher Superstition* (Baltimore, MD: The John Hopkins University Press, 1994), 93.

[20] Ralph Abraham and Yoshisuke Ueda, *Chaos Avant-Garde: Memoirs of the Early Days of Chaos Theory* (Singapore: World Scientific Publishing Co. Pte. Ltd., 2000), 86.

[21] The meteorologist Edward Lorenz created the *Lorenz attractor* by presenting the trajectories of three coupled non-linear differential equations.

for the first time, Chaos Theory was brought to the attention of the international physics community."[22] That meeting brought people out of isolation and into communication with each other. Gleick stresses that by the middle of the 1980s scientific centers and institutes had been founded to specialize in nonlinear dynamics and complex systems. He argues that "Chaos had become not only a theory or a canon of beliefs but also a method and a way of doing science."[23]

2. Randomness, Chaos, and Chaos

The second body of literature deals with the confusion that the term Chaos usually causes to theorists of the social sciences. Before starting to recognize the action of Chaos in IR, we have to keep in mind two major distinctions, the first between randomness and chaos, and the second between "Chaos" and "chaos."[24]

Chaotic change is distinct from "random" change. To use the mathematical tools that CT offers, we have to distinguish between Chaos and randomness. As Chaos is a type of non-random behavior, we must first explain the difference between randomness and non-randomness. Robert W. Batterman gave a very simple example where he suggests comparing two infinite sequences of symbols, which are numbers:[25]

1st seq. 0101010101010101010101010101….. (Non-random)

2nd seq. 1100111000110000110110010011101 1….. (Random)

As Glenn maintained, in this example of the non-random case, if we have "access to only a brief list of the first few elements of the sequence" we could conclude about its behavior.[26] But for the random case, we need the entire infinite string to define the

---

[22] Ralph H. Abraham, "The Chaos Revolution: a Personal View," *Chaostory* Rev 3.1, based on a lecture at Kyoto University, March 20, 1998.

[23] Gleick, *Chaos Making a New Science*, 38.

[24] Paul R. Gross and Norman Levitt, *Higher Superstition* (Baltimore, MD: The John Hopkins University Press, 1994). Gross and Levitt, with their book *Higher Superstition,* express the opinion that the social scientists cannot use CT as they have not mathematical background.

[25] Robert W. Batterman, "Defining Chaos," *Philosophy of Science*, vol. 60, no. 1 (1993): 43–66.

[26] Glenn, *Chaos Theory: The Essentials for Military Applications*, 107.

second sequence. That is the simplest way to explain the difference between non-randomness and randomness. To be more precise, with some known elements within a system with chaotic behavior, prediction is possible. Even with a huge amount of data about a system with random behavior, no prediction is possible.

A second distinction that we have to keep in mind in order to study IR through the lens of Chaos is the difference that Glenn introduces between "Chaos" and "chaos." Glenn argues that "Chaos" with a capital C is a mathematical discipline with boundless applications, and it has no relation to social disorder, anarchy, or general confusion; [27] on the other hand, "chaos" is the well-known social chaos related to negative situations like conflicts, wars, and disasters. He explains that for natural scientists Chaos is a tool "to recognize the unstable orbits embedded within a chaotic attractor" in a dynamical system.[28] To say that Chaos works, social scientists should be able to imitate physicists to produce models that will indicate the existence of Chaos, which in turn can inform adjustments in policy by decision makers. Is it possible?

Despite the complexity of behavior within dynamic systems, there have been great improvements by experimental physicists in controlling a chaotic system. Edward Ott and Mark Spano argue that the "orbital complexity and exponential sensitivity of chaotic systems" enable such systems to be feedback controlled using small perturbations. They maintain that "the potential consequences of this realization are being investigated in a broad range of applications" such as simple mechanical systems, electronics, chemical systems, and heart or brain tissues.[29] Such research has inspired many social scientists to believe that they can identify and control Chaos in IR.

Glenn stresses that "Chaos is not hard to learn, it is only hard to learn quickly" and that it is essential for everybody because we may fail to recognize chaotic phenomena in our physical and social systems if we are not familiar with them.[30] He also

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[27] Glenn , *Chaos Theory: The Essentials for Military Applications*, 2.

[28] Ibid.

[29] Ott and Spano, "Controlling Chaos," 92–103.

[30] Glenn, *Chaos Theory: The Essentials for Military Applications*, xii.

argues that the applications of Chaos "are so extensive that decision makers need to be familiar with Chaos Theory's key results and insights."[31]

3. Chaos Theory and International Relations

The third body of literature focuses on the use of CT by IR theorists. Political scientist Dylan Kissane suggests that three assumptions about international behavior allow us to extract a series of predictions about the international system in the short and medium terms.[32] The first of these assumptions is that the nature of the international system is Chaotic, which means the international system is by nature sensitive to initial conditions, complex long-term behavior, and unpredictability. The second is that actors in a system with chaotic behavior seek security. The third assumption is that, the need for security makes the actors to interact. According to Kissane's assumptions, we may handle and control such a system by imitating physicists. The problem with this assumption is that every notion is abstract and not measurable and the variables that can affect the system are not defined; for a system to be classified as chaotic we need exactly to define the system, the system's differential equations, and the system's variables. The expression 'the international system is chaotic' is a vague expression if we try to explain it with the physical term of Chaos, but it makes sense in relation to social chaos that I described previously.

Manuel Ferreira et al. stress some examples of application areas for Chaos in politics.[33] In this context, public organizations may be examined as dynamical systems and their actions analyzed by studying their operational stability. The study of peace scenarios using the tool of Chaos Theory focuses, according to Ferreira et al., on "the relation between order and disorder in the emergence of peace."[34] Political parties and elections can also be viewed through a chaotic approach because some minor events of an

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[31] Glenn, *Chaos Theory: The Essentials for Military Applications*, xii.

[32] Kissane, "A Chaotic Theory of International Relations?" 85–103.

[33] Manuel A. M. Ferreira, Filipe J. A. C. Bonito, Manuel. F. P. Coelho, and Isabel C. Pedro, *Chaos Theory in Politics* (London: Springer Editions, 2014), 95.

[34] Ibid.

electoral campaign are able to change completely the final results; that is sensitivity to initial conditions.

Many other specific case studies that contain chaotic phenomena exist, such as the Iranian Revolution of 1978–1979, the rise of Adolf Hitler in Germany, Alexander's conquest of the Persian Empire, the arrival of Attila to Europe, the onset of the two Gulf Wars, the Arab Spring, and the 9/11 attack in the United States. The common characteristic underlying these events is sensitivity to initial conditions. For the previously mentioned cases the chaotic characteristic is that some minor event caused a huge disaster—mainly involving the loss of human life—which the authors describe as a chaotic situation. All these situations can be described with mathematical models; however, it is essential first to define each possible parameter that could affect the system of each case.[35]

Ferreira et al. also argue that the "inherently nonlinear phenomena present in politics indicate that it is possible to use mathematical models in the analysis of the political environment" and socio-political issues such as the aforementioned examples.[36] Nevertheless, Ferreira et al. do not define what mathematical models are applicable and how are they related to applying CT to politics. The application of a mathematical model works for well-defined systems with specific laws and equations.[37]

On the other hand, there are scholars, like Harmke Kamminga, who consider using mathematical theories, like CT, to explain politics inappropriate.[38] Kamminga characterizes human social systems as important but highly problematic in terms of how to define and analyze them. He states that "the construction of chaotic mathematical models of real systems involves important simplifications, which could have enormous consequences for our understanding of real dynamical systems; models are theoretical

[35] The international system cannot be considered as an integrated system like the physical system, because we cannot define the laws, the equations, and the actions that affect it.

[36] Ferreira et al., *Chaos Theory in Politics,* 95.

[37] Glenn, *Chaos Theory: The Essentials for Military Applications*, xiv.

[38] Harmke Kamminga, "What Is This Thing Called Chaos?," *New Left Review I,* no.181 (May–June 1990), 52.

constructs which are intended to capture key features of real systems."[39] They represent ideal situations, so the construction of good realistic models is for Kamminga meaningless. Real systems are always to some extent open while CT is very specific without simplifications. Yet, Saperstein's comment is the answer to Kamminga's concerns:

> Dynamical modeling is an important component of verbal political science. "Modeling" refers to the creation of a representation of the world of interest—in your mind, on paper, or in a laboratory. You cannot incorporate the entire real world in your mind whereby you can manipulate it so as to attain understanding of its dynamics (change some aspect of it and see how the rest changes). Hence you use a representation, a partial world which you hope, contains the aspects of that world important for the behavior that you wish to understand. Whether or not that hope is justified will be determined by subsequent testing in the real world, of the results of your understanding of the model world. Again, modeling is a necessary characteristic of conventional political science, though usually informally and implicitly.[40]

In the same vein with Kamminga, Gross and Levitt raise serious questions about the implementation of Chaos Theory by humanists and social scientists. They point out that these analyses in effect undermine the reliability and accuracy of standard science.[41] They argue that the popularizations of some books have the effect of deceiving the "intelligent layman" into believing that he grasps the subject better than he really does. For them a solid understanding of what is really involved requires a considerable amount of formal mathematical knowledge.[42]

A moderate solution for the use of CT in IR comes from Saperstein who is among the first scientists to apply Chaos Theory to social sciences. According to Saperstein, nonlinear dynamical systems theories deal with mathematical models, so the implementation of CT in IR could be done with models. We cannot use the models as the solution to everything, and uncritical use can cause misguided predictions. Real world

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[39] Ibid.

[40] Saperstein, *Dynamical Modeling of the Onset of War*, 10.

[41] Gross and Levitt, *Higher Superstition*, 92.

[42] Ibid.

systems are open, without well-defined boundaries. The complexity of real world systems makes impossible the collection of all the necessary information to create a model system of equations adequate to predict the future of IR among all states. Even so, examining isolated sub-systems of the real world, which are limited to a small number of variables, could be helpful for the decision makers.[43] This solution satisfies the demands of physicists—by being mathematically sound—and it can be applied to topics of concern in IR.

D. POTENTIAL EXPLANATIONS AND HYPOTHESES

Chaos Theory is a tool that aims to provide scientists with the option to observe and finally control the chaotic phenomena. Chaotic and complex phenomena are two notions that indicate the human inability to understand and control such phenomena. Initially in physics and later in IR, scientists used the theory with several results.

This thesis formulates three hypotheses:

1. CT does not work for physics as physicists have not yet achieved control of chaotic phenomena. In this case, all efforts to apply the theory in IR issues are meaningless.
2. CT works for physics, but it does not for IR. In this case, the results of Saperstein's model are irrelevant to conventional theories or they are biased because they are close to one conventional theory only.
3. CT works both for physics and IR. In this case, the results of Saperstein's model are relevant to more than one conventional theory of IR, and CT can be considered universal and beneficial for both domains.

E. METHODOLOGY

The method of analysis in this thesis is the comparative analysis of two different domains to answer whether CT is a universal theory. It incorporates the qualitative and

[43] With the term sub-system I mean a part of a system or the whole system in isolated conditions. For instance, when we use some economics models to predict the economic behavior of a country we do not take into account that this economy is in danger of an earthquake or a tsunami that could destroy it.

quantitative study of CT in physics, and through the use of the OGY method, it checks the physicists' ability to predict and control for Chaos; that will answer whether CT can be utilized in physics to achieve control of chaotic phenomena.[44] For the IR domain, the Saperstein model, in combination with conventional theories, will answer the question of whether CT can be utilized in the IR field. Toward this end, this research uses primary sources, such as empirical studies—research where an experiment was performed or a direct observation was made, as well as secondary sources of literature, such as books and peer-reviewed articles, which discuss, interpret, and analyze the Chaos Theory, IR theories, and the link between Chaos Theory and political science.

F. THESIS OVERVIEW

This thesis is divided into six chapters. The first chapter presents the research question, the importance of this study, the literature review of the topic, the potential explanations and hypotheses, and the construction of this thesis.

The second chapter introduces the reader to the concept of Chaos Theory. With the help of a simple pendulum—where I add constraints that transform it into a driven damped pendulum (DDP)—I explain what represents the theory of Chaos. From the simple pendulum I move my analysis to the damped pendulum and finally to the DDP, which is the simplest chaotic system. I present the basic tools that physicists use to study Chaos so as to familiarize the reader with notions such as attractor, phase-space diagram, and nonlinearity. I explain the equations that describe the DDP, and I analyze two cases: the first is the case of the DDP with a small driving force, and the second with a bigger one that causes Chaos.

The third chapter analyzes the controllability of Chaos in physical systems. I start with physicists' exploitation of chaotic behavior and how that changed the perception that chaos is undesirable. Then, I describe the OGY method that was the first method to

[44] The OGY method took its name from the initial letters of Ott, Grebogi, and Yoke; the physicists who applied it first.

control chaos. Finally, I present applications that prove that Chaos controllability exists and that physicists are able to predict, track, and control Chaos.

The fourth chapter analyzes the two major conventional theories for IR analysis, Realism and Liberalism. First, I explain the content of these theories; then, I describe their historical orientation, and finally, I compare them through several different frameworks (political, IR, economic).

The fifth chapter contains a comparison between the results of Saperstein's chaotic model and the conventional theories of IR. First, I describe the two IR puzzles that will be modeled—bipolarity and democratic peace—and I describe the model, then I review the Saperstein results on two different IR questions, and finally, I compare the results of the chaotic model with conventional theories.

The final chapter summarizes the findings and proposes some ideas for future research.

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II. THE PHYSICS APPROACH TO CHAOS THEORY

A. EXAMPLES OF CHAOTIC SYSTEMS

The simplest and most common chaotic system is the driven damped pendulum, or DDP. To understand this system we begin with the easily understood ideal pendulum, then explain the damped pendulum, and finally define and review the DDP. Glenn explains that "an extraordinary number of complicated physical systems behave just like a pendulum or just like several pendulums that are linked together."[45] The DDP will introduce us to the nature of physical dynamical systems in order to understand how they behave and to question whether it is possible to implement Chaos Theory on social dynamical systems and especially those which are related to international relations.

Two common characteristics of chaotic behaviors are exponential sensitivity and orbit complexity, which are typical of systems that move or change, such as the DDP or weather models for prediction. When trying to apply CT to a given system, the first step is to define the system in question. In defining the system we identify two things: a collection of elements and the rules that the elements obey.[46] Specifically, the collection of elements refers to the components, players, or variables that make up the system. The set of rules concerns formulas, equations, recipes, or instructions which govern the system.[47] The most common confusion results from failing to accurately identify the system or its variables.

1. The Ideal Pendulum

The system of an ideal pendulum, the so-called simple pendulum (Figure 1), is a weight—a bob of mass—suspended from a pivot so that it can swing freely. To put it another way, it is a mass that is fixed on a massless rod, which pivots at a point and is free to swing without friction or air resistance in the vertical plane. Glenn describes that

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[45] Glenn, *Chaos Theory: The Essentials for Military Applications*, 10–15.

[46] Ibid., 3.

[47] Ibid.

the bob moves in two dimensions, so in this system "we need only two pieces of information to completely describe the physical state of the system: *position* and *velocity*." These two observable quantities are often referred to as phase variables. The bob is moving in the vertical plane, and therefore, its location needs to be defined by two coordinates. However, the trajectory is constrained from the rod; as a result, we need only the angular position of the mass to have the position of the pendulum. As only one of the coordinates is independent, we say that the system has only one degree of freedom.[48]

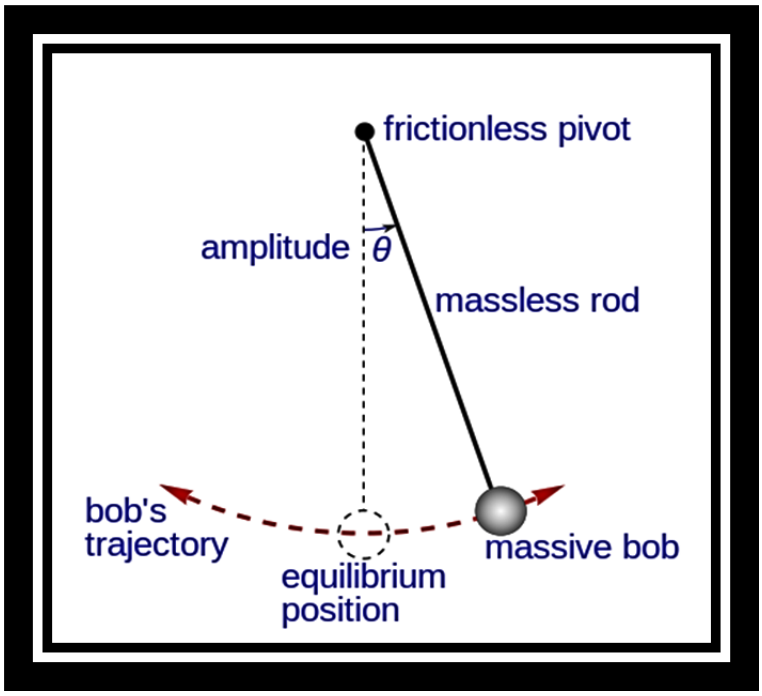

Figure 1. Simple Pendulum.[49]

The motion of this pendulum is an idealization of the operation of a pendulum in an isolated system, with no external forces except for gravity.[50] We can interact with this system in several ways. If we place the bob at the lowest point of the trajectory it stays at

[48] Taylor, *Classical Mechanics*, 245–246.

[49] Adapted from Wikipedia the free encyclopedia, "Pendulum," https://en.wikipedia.org/wiki/Pendulum .

[50] In Newtonian mechanics, gravity is the force of attraction between two masses.

this equilibrium position forever and this is called stable equilibrium.[51] If we displace the bob and let it go it will move forever as the only force will be its weight. The time for one complete cycle of the bob after it is released from a given point (a left swing and a right swing), is called the period and this is constant for this system.[52]

To understand the operation of the ideal pendulum under the conditions of the real world, we have to add some resistance. As a result, this force will damp the pendulum (by forcing it to lose its potential energy),[53] which slows its swing and returns the pendulum to its equilibrium position.[54] The swinging action eventually stops due to resistance in the environment, called transient dynamics. As the friction dissipates the system's energy, the mass comes to rest at the central fixed point.[55]

Physicists wanted a "map" to describe the behavior of dynamical systems so they found a way to turn numbers into pictures.[56] For our pendulum, the two variables, position and velocity, define the state of the system, and the space of the system is called the phase space (or state space). For any system, this is a space in which all the possible states are represented and every bit of essential information is abstracted. Glenn describes that "every degree of freedom or parameter is represented as an axis of multidimensional space and the points trace a phase-space trajectory that provides a way of visualizing the long-term behavior of dynamical system."[57]

Gleick states that "for a pendulum steadily losing energy to friction, all trajectories spiral inward to a point that represents a steady state, especially in this case the steady state of no motion at all" (Figure 2).[58] We call this point an *attractor* of the

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[51] Taylor, *Classical Mechanics*, 245–246.

[52] Ibid.

[53] Potential energy is the energy of a body with respect to its position.

[54] Glenn, *Chaos Theory: The Essentials for Military Applications*, 12.

[55] Ibid.

[56] Ibid., 134.

[57] I use the "phase space" definition as it is used by Thierry Vialar in his book *Complex and Chaotic Nonlinear Dynamics: Advances in Economics and Finance, Mathematics and Statistics* (Berlin: Springer, 2009).

[58] Gleick, *Chaos Making a New Science*, 138.

system and it is a single point; the attractors exist in the phase space, and they are one of the most powerful inventions of modern science. The concept of an attractor reflects how all the states of a system, corresponding to different initial conditions, are attracted to a specified and pre-determined final state. Hence, if we have found the attractors of a dynamical system we can predict the long-term behavior of that system, even if it is very sensitive to the initial conditions.

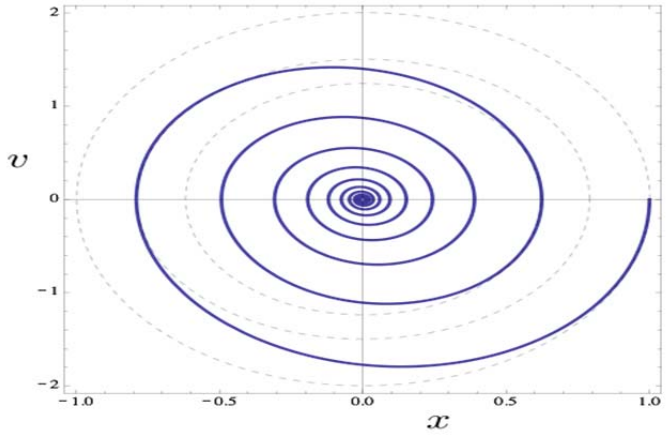

Figure 2. Phase-Space Trajectory of the Damped Pendulum.[59]

2. The Driven Damped Pendulum

We started from the ideal pendulum, jumped to the damped pendulum, and to add the last essential layer of reality, we add a small driving external force.[60] This system is the driven damped pendulum that shows chaotic characteristics. For a DDP with a small sinusoidal driving force, there is a unique attractor, which the motion approaches, irrespective of the chosen initial conditions. The drive strength increases the nonlinearity of the pendulum's motion, which starts to diverge from the periodic motion. When nonlinearity is dominant, different initial conditions can lead to totally different attractors, and this is the fundamental action of Chaos.

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[59] Source: Alexei Gilchrist, "Dynamical System Maps," accessed September 17, 2015, http://www.entropy.energy/scholar/node/dynamical-systems-maps.

[60] This driving external force moves the pendulum with small "kicks" and specific frequency.

With DDP we can find many useful applications of physics as Chaos exhibits extreme sensitivity to initial conditions. John R. Taylor states that this sensitivity is what can make "the reliable prediction of chaotic motion a practical impossibility."[61] By differentiating the choices of driving amplitude, driving frequency, or damping, the DDP will produce different behaviors in the long term. With small driving amplitude the pendulum will behave as a damped harmonic oscillator, but with a weak damping the pendulum will be a driven oscillator with a period equal to the driving frequency.[62] Keeping other parameters constant, but increasing the driving frequency, can produce a new period of oscillation, which is named the *period two oscillation* (half driving frequency) while the period one oscillation becomes unstable.[63] The continuation of this increasing produces period four, period eight, and onward, and the bifurcations come faster and faster until the period of oscillations is infinity. At this point in time, there is no stability; Chaos starts![64]

B. THE DRIVEN DAMPED PENDULUM'S EQUATIONS

To Shakespeare's question "what's in a name?" the physicists William L. Ditto et al. answer "nothing and everything." They explain "nothing [by citing Shakespeare] because 'a rose by any other name would smell as sweet.' And yet, without a name Shakespeare would not have been able to write about that rose to distinguish it from other flowers that smell less pleasant. So also with chaos." In 1975, James Yorke gave the name Chaos to define exactly what a chaotic behavior is; the equations of a DDP obey what Yorke defined as chaotic. This chapter presents the mathematical concept of a DDP's motion that determines a chaotic behavior.

While the equations and the numbers are the main tool for physicists to have results in a research, the French mathematician Henri Poincare proposed a new analysis

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[61] Taylor, *Classical Mechanics*, 480.

[62] In Chapter III, I explain this procedure analytically.

[63] Taylor, *Classical Mechanics*, 481–485.

[64] Robert M. May, "Simple Mathematical Models with Very Complicated Dynamics," *Nature* 261 (1976): 459–467.

technique using qualitative analysis. Instead of looking at the trajectories as functions of time, he tried to answer these questions: "Is Solar System stable?" "Is there any Planet that can be out of Solar influence by a time interval?" "Can any planetary trajectory go to infinity?" Luiz F.R. Turci et al. argue that Poincare to answer these questions developed powerful geometric methods.[65] In the last 20 years, scientists have developed techniques that track Chaos after the confirmation of its existence; however, they use equations and numbers to confirm that there is a chaotic behavior; qualitative analysis is not enough.

In the previous sections, I explained qualitatively the chaotic behavior of a DDP by imitating Poincare's method; now I will explain, with the help of the DDP (depicted in Figure 3) and its equations of motion, how to confirm the existence of Chaos.

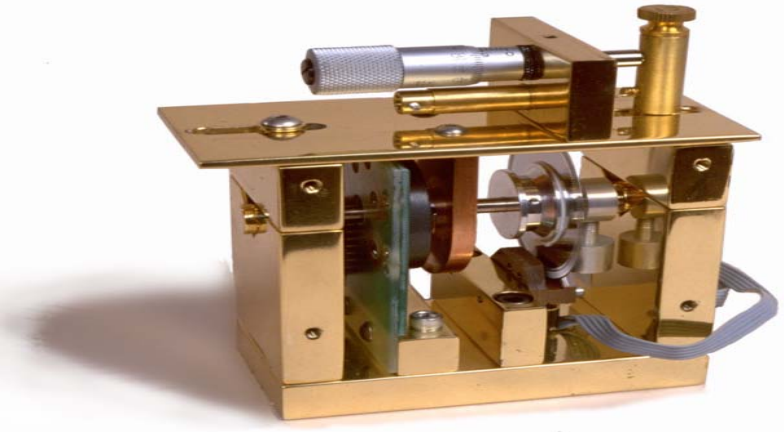


Figure 3. Driven Damped Pendulum Apparatus.[66]

The equation of motion for the DDP is

$$I\ddot{\varphi} = \Gamma, \tag{1}$$

where I is the moment of inertia and Γ is the net torque about the pivot. We also have that

$$I = mL^2. \tag{2}$$

[65] Luiz F. R. Turci and Elbert E. N. Macau, "Chaos Control," in *Recent Progress in Controlling Chaos*, ed. Miguel A. F. Sanjuan and Celso Grebogi (Singapore: World Scientific Publishing, 2010), 373.

[66] Source: Fraden Group, "Driven Damped Pendulum Apparatus," accessed October 22, 2016, http://fraden.brandeis.edu/courses/phys39/chaos/chaos.html.

The torque is the result of the torques from the resistive force, the driving force, and the weight

$$\Gamma = \Gamma_R + \Gamma_W + \Gamma_D . \tag{3}$$

The aforementioned torques are

$$\Gamma_R = -Lbv = -bL^2\dot{\varphi} \quad (4), \qquad \Gamma_w = -mgL\sin\varphi , \tag{5}$$

$$\Gamma_D = LF(t) . \tag{6}$$

For a sinusoidal force, we have that $F(t) = F_o \cos\omega t$ so the relation (6) is $\Gamma_D = LF_o \cos\omega t$ (7). By substituting the relations (2), (3), (4), (5), (7), to the relation (1) we have

$$mL^2\ddot{\varphi} = -bL^2\dot{\varphi} - mgL\sin\varphi + LF_o\cos\omega t \Leftrightarrow \ddot{\varphi} + \frac{b}{m}\varphi + \frac{g}{L}\sin\varphi = \frac{F_o}{mL}\cos\omega t . \tag{8}$$

We also have that

$$\frac{b}{m} = 2\beta , \tag{9}$$

where β is the damping constant,

$$\frac{g}{L} = \omega_o^{\,2} , \tag{10}$$

where ω_o is the natural frequency, and

$$\gamma = \frac{F_o}{mg} , \tag{11}$$

where γ is the drive strength. The dimensionless parameter γ is indicative of the magnitude of the driving force. For γ<1 we have a small motion while for $\gamma \geq 1$ we have a force bigger than the weight, which produces larger scale motions. Now, the relation (8) with the relations (9), (10), (11) is

$$\ddot{\varphi} + 2\beta\dot{\varphi} + \omega_o^{\,2}\sin\varphi = \gamma\omega_o^{\,2}\cos\omega t . \tag{12}$$

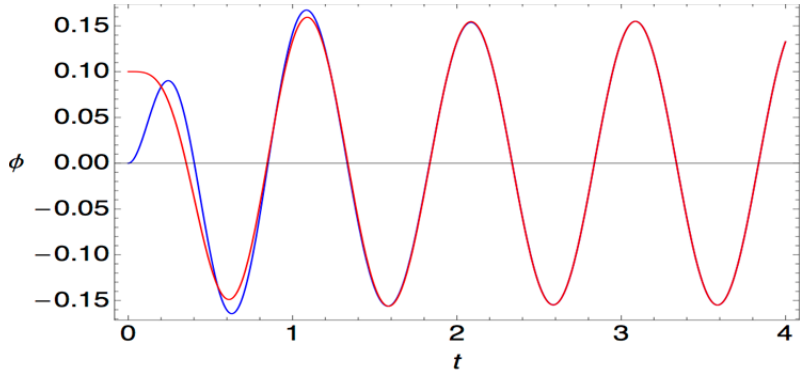
The previous equation has the form that Taylor proposes to achieve a chaotic motion; it is "nonlinear and somewhat complicated."[67]

C. THE CASE OF THE DRIVING FORCE SMALLER THAN THE WEIGHT, Γ<1

With the initial conditions $\varphi = 0$ $\dot{\varphi} = 0$ at $t = 0$ and $\varphi = 0{,}1$, $\dot{\varphi} = 0$, at $t = 0$ and a very weak drive strength, where $0{,}1 \le \gamma \le 0{,}2$, we have the properties of the linear oscillator (Figure 4). We can approximate that $\sin\varphi \approx \varphi$ and the relation (12) will be

$$\ddot{\varphi} + 2\beta\dot{\varphi} + \omega_o{}^2\varphi = \gamma\omega_o{}^2 \cos\omega t \,,$$

which is a linear equation.[68] With the (φ, t) diagram and the $(\varphi, \dot{\varphi})$ phase-space diagrams (Figure 5 and 6), we can see that there is a unique attractor for different initial conditions of φ.



The two curves start from different values of φ, but finally they converge.

Figure 4. (φ, t) Diagram with Weak Strength $\gamma = 0.1$ and Different Initial Conditions. [69]

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[67] The source for all the previous equations is John. R. Taylor, *Classical Mechanics* (Colorado: University Science Books, 2005), 460–463.

[68] Taylor, *Classical Mechanics*, 464–465.

[69] Source: University of Colorado, "(φ, t) diagram," accessed October 22, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

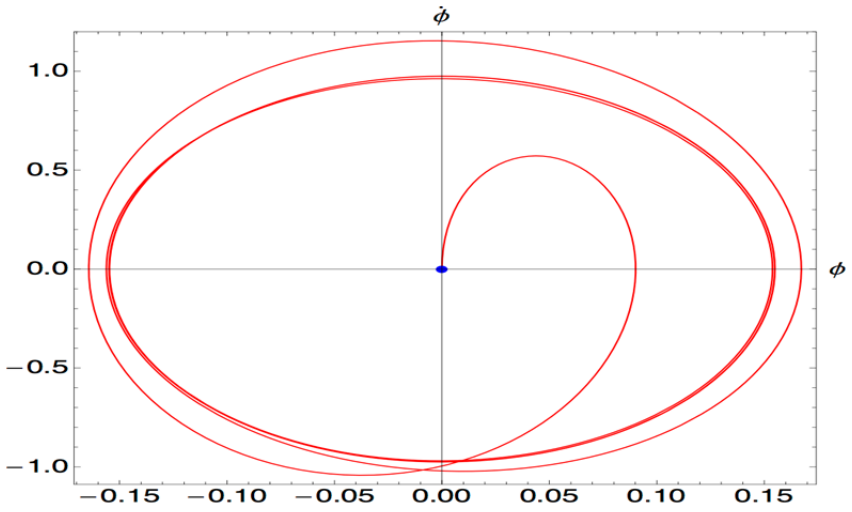


Figure 5. Phase-Space Diagram $(\varphi, \dot{\varphi})$ with Weak Strength $\gamma = 0.2$ and Initial $\varphi = 0$.[70]

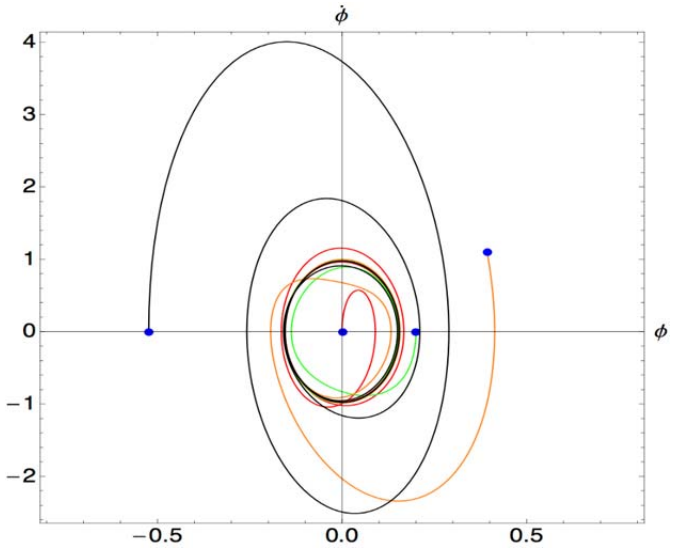


Figure 6. Phase-Space Diagram $(\varphi, \dot{\varphi})$ with Weak Strength $\gamma = 0.2$ and Initial $\varphi = 0, 1$.[71]

[70] Source: University of Colorado, "$(\varphi, \dot{\varphi})$ diagram," accessed October 22, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

[71] Source: University of Colorado, "$(\varphi, \dot{\varphi})$ diagram," accessed October 22, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

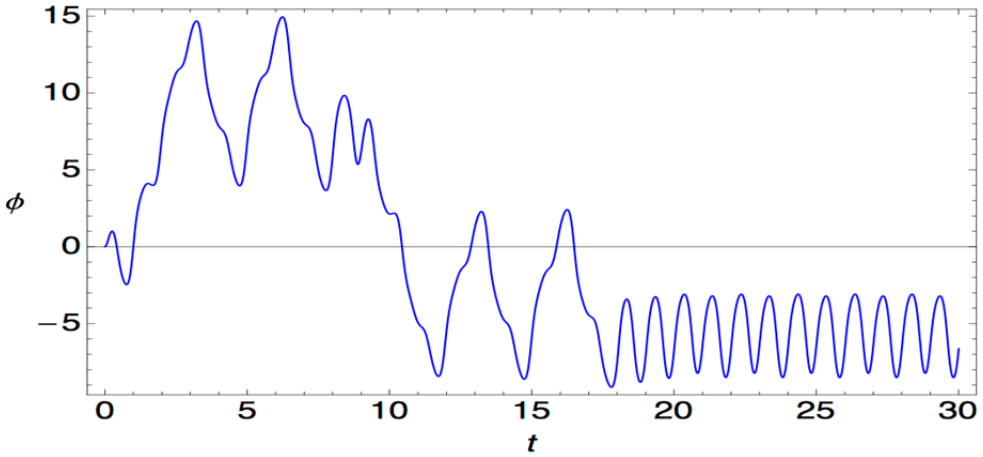
In the case of $\gamma = 0.9$, we will have again a convergence after the initial transients die out, and the frequency is called the *harmonic* of the drive frequency as the pendulum's motion will have a frequency equal to an integer multiple of ω. As Taylor notices "in the linear regime the motion is given by a simple cosine" while " in the not-quite-linear regime (γ somewhat larger, but definitely not much greater than 1), the motion picks up some harmonics."[72]

D. THE CASE OF THE DRIVING FORCE GREATER THAN THE WEIGHT, Γ>1

By increasing the force to $\gamma = 1.06$, we have the pendulum to move similarly with $\gamma = 0.9$. That happens after a strange oscillation of the initial transient motion. The pendulum starts a wild oscillation, but after 35 cycles we have a motion that approaches an attractor that has the same period as that of the driver. The initial conditions are $\varphi(0) = 0, \dot{\varphi}(0) = 0$.

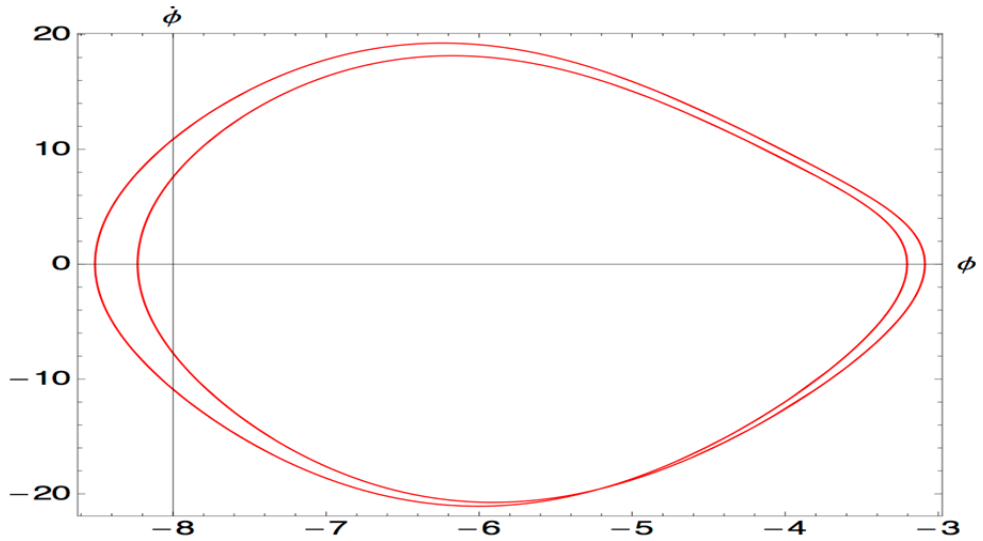
For the same initial conditions and with a little bit more strength, $\gamma = 1.07$ we also have a wild initial oscillation (Figure 7). However, after 20 cycles the motion becomes periodic (Figure 8), but with double period compared with the drive period. So, we say that the motion has period two.

[72] Taylor, *Classical Mechanics*, 467.



After the initial transient motion, the system settles down into a simple periodic behavior.

Figure 7. The (φ, t) Diagram for $\gamma = 1.07$.[73]



The phase-space trajectory after the initial transient motion indicates the periodic motion.

Figure 8. The $(\varphi, \dot{\varphi})$ Diagram for $\gamma = 1.07$.[74]

[73] Source: University of Colorado,"(φ, t) diagram," accessed October 27, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

[74] Source: University of Colorado, "$(\varphi, \dot{\varphi})$ diagram," accessed October 27, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

For drive strength $\gamma = 1.077$, after the initial transients die out, the motion has period equal three times the drive period. For the initial conditions $\varphi(0) = 0, \dot{\varphi}(0) = 0$ the attractor repeats itself every three drive cycles; however, for different initial conditions we have different attractors. Taylor stresses that for $\varphi(0) = \frac{-\pi}{2}, \dot{\varphi}(0) = 0$ we have an attractor with period two, so he concludes that "for a nonlinear oscillator different initial conditions can lead to totally different attractors."[75]

The diagram in Figure 9 shows that for different initial conditions we will have different attractors; however, there is something common with the previous initial conditions, the *period doubling cascade*. On the left the distinction is not so obvious, but with the enlargement on the right, it is clear that there is a significant difference. This doubling cascade happens for specific values of γ, which are related with the initial conditions. The γ values that double the period are called *threshold values* or *bifurcation points*.[76]

[75] Taylor, *Classical Mechanics*, 467–71.

[76] Taylor, *Classical Mechanics*, 471–474.

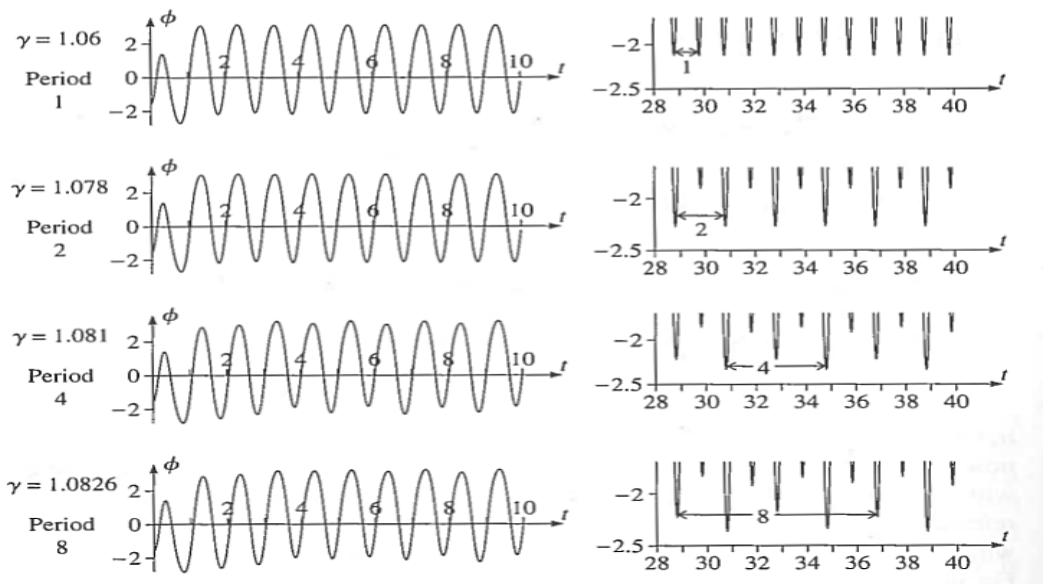


Figure 9. The Period Doubling Cascade.[77]

In my first example, period two happened for $\gamma_1 \approx 1,07$, period three for $\gamma_2 \approx 1,077$, and if we continue we can find the values for period four and so on. After the setting of initial conditions we can find different driving forces that can double the period of the pendulum and this is a unique phenomenon that sets the route to Chaos. Following these results, in the 1970s the physicist Mitchell Feigenbaum proposed the relation

$$(\gamma_{n+1} - \gamma_n) \approx \frac{1}{\delta}(\gamma_n - \gamma_{n-1}),$$

where $\delta = 4,6692016$, and it is called the Feigenbaum number. According to the aforementioned relation the γ_n tends to be stable as $n \to \infty$ to a finite number; the critical value γ_c. The importance of the γ_c value is that for γ greater than γ_c we have long-term erratic, non-periodic motion. The motion of the pendulum cannot converge to a stable period, and this is what we call chaotic motion.[78]

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[77] Source: Taylor, *Classical Mechanics*, 472.

[78] Taylor, *Classical Mechanics*, 471–474.

By changing the initial conditions we also saw that the pendulum's motion changed dramatically. This is the second characteristic of the chaotic motion; the sensitivity to the initial conditions. The physicists Gregory Baker and Jerry Gollub describe this phenomenon:

> The fundamental characteristic of a chaotic physical system is its sensitivity of the initial state. Sensitivity means that if two identical mechanical systems are started at initial conditions x and $x+\varepsilon$ respectively, where ε is a very small quantity, their dynamical states will diverge from each other very quickly in phase space, their separation increasing exponentially on the average.[79]

Two identical DDPs can diverge dramatically by changing only a fraction of a degree for the initial φ of them. At the chaotic regime of DDP's motion for $\gamma = 1,105$ we start the two pendula with an initial separation of $\Delta\varphi(0) = 0,0001$ and the two diagrams diverge quickly (Figure 10).

[79] Gregory L. Baker and Jerry P. Gollub, *Chaotic Dynamics: An Introduction,* 2nd ed. (New York: Cambridge University Press, 1996), 42.

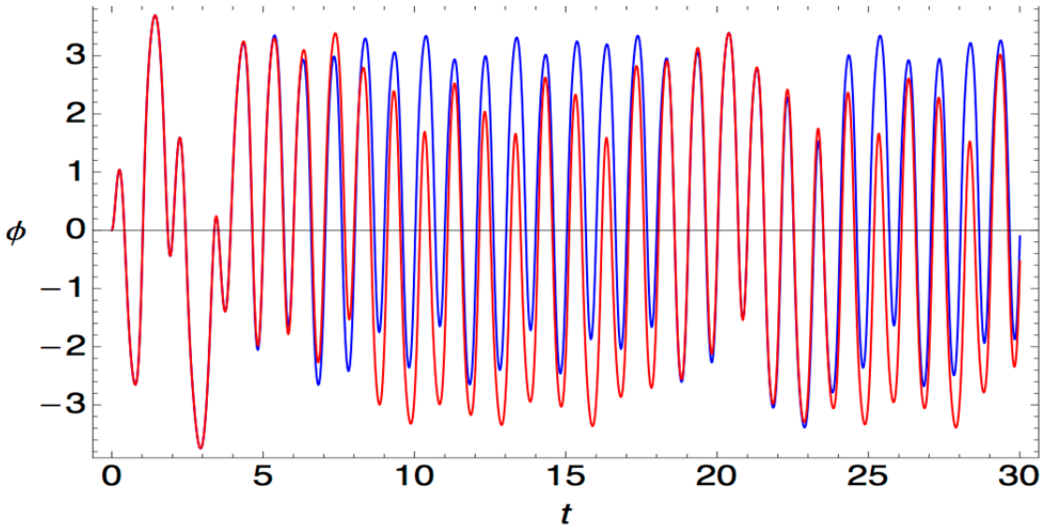

Figure 10. The (φ, t) Diagram for Two Identical DDPs with $\Delta\varphi(0) = 0,0001$.[80]

The difference between the two DDPs is even better illustrated with the $(\log|\Delta\varphi(t)|, t)$ diagram (Figure 11). The separation grows exponentially over time and the chaotic motion of the pendula is present. The relation that expresses this growth is

$$|\Delta\varphi(t)| \approx Ke^{\lambda t}.$$

The coefficient λ is called the *Liapunov exponent*, and the positive values of λ express the long-term chaotic motions.[81]

[80] Source: University of Colorado "(φ, t) diagram," accessed October 27, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

[81] Taylor, *Classical Mechanics*, 480.

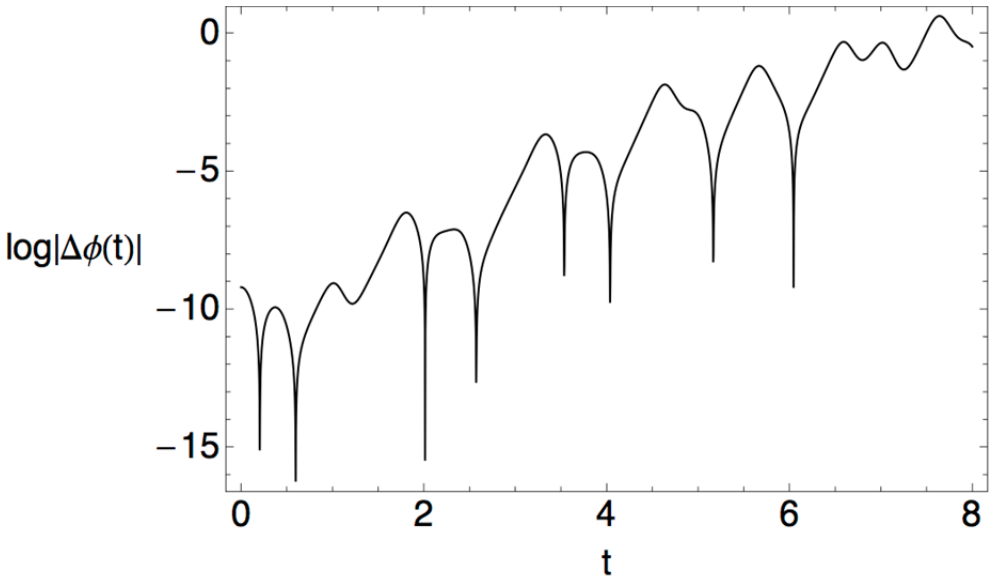

Figure 11. The $(\log|\Delta\varphi(t)|, t)$ Diagram for Two Identical DDPs with $\Delta\varphi(0) = 0,0001$.[82]

The last tool that we need to examine the existence of Chaos is the bifurcation diagram (Figure 12). In the previous paragraphs, we saw examples and diagrams for different initial conditions, but the values of γ were stable. The bifurcation diagram presents the changing of the $\varphi(t)$ through the increase in the value of γ. For the DDP, the bifurcation diagram shows the period-doubling cascade and the existence of Chaos after the critical value $\gamma_c = 1.0845$; for each value of γ, there are hundred values of $\varphi(t)$, which are indicated by dots on the diagram in Figure 12.

[82] Source: University of Colorado, "$(\log|\Delta\varphi(t)|, t)$ diagram," accessed October 27, 2016, http://www.colorado.edu/physics/phys3210/phys3210_sp14/lecnotes.2014-04-18.Nonlinear_Dynamics_and_Chaos.html.

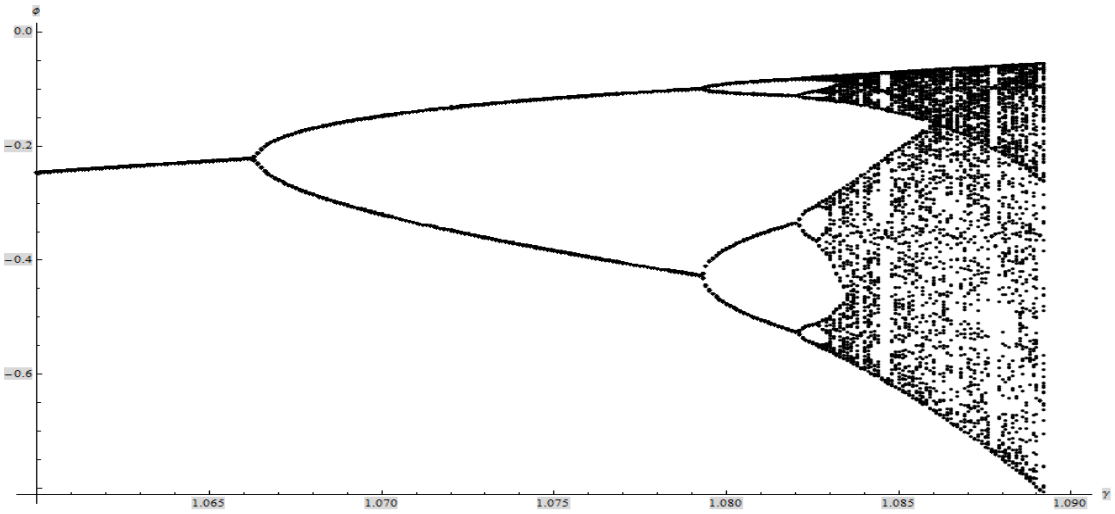

Figure 12. The Bifurcation Diagram of a Driven Damped Pendulum.[83]

[83] Source: Mathematics Stack Exchange, "The Bifurcation Diagram of a Driven Damped Pendulum," accessed October 27, 2016, http://math.stackexchange.com/questions/380310/chaos-without-period-doubling.

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III. CONTROLLING CHAOS

The Dutch artist M. C. Escher stated that "we adore chaos because we love to produce order."[84] This statement may hide the desire that the physicists have had all these years to control the chaotic dynamics that were present and which were undesirable. In 1975, the physicists Y. Li and J. Yorke introduced the term "Chaos" with their article "Period Three Implies Chaos." It took around 15 years for the physicists E. Ott, C. Grebogi, and Yorke to present their attempt to "produce order." The method known as OGY (from the initial letters of their surnames) involves the exploitation of the changes that can cause a small number of perturbations to a chaotic system. William Ditto et al. argue that the OGY method is the response to the question: "if a system is so sensitive to small changes, could not small changes be used to control it?"[85] The OGY method inspired physicists to develop several techniques to control chaotic phenomena, and they succeeded in exploiting chaos by manipulating it. This chapter presents the advantages of a chaotic behavior, the OGY method to control chaotic dynamics, and the applications of Chaos controllability.

A. ADVANTAGES OF CHAOS

The features of chaotic dynamics, being sensitive to initial conditions and having orbit complexity, make them undesirable to physicists. The inability to predict the behavior of a system over a long time because of their exponential growth is a disadvantage that the physicists turned into an advantage. Ditto et al. describe it as: "paradoxically, the cause of the despair is also the reason to hope."[86] In the same vein, the physicists Mark Spano and Edward Ott argue that Chaos is advantageous in many cases and for the situations that it is "unavoidably present, it can often be controlled and

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[84] M. C. Escher quotation, accessed November 20, 2016, http://www.goodreads.com/author/quotes/306401.M_C_Escher.

[85] Ditto et al., "Techniques for the Control of Chaos," 199.

[86] Ditto et al., "Techniques for the Control of Chaos," 199.

manipulated to obtain the desired results."[87] In a periodic attractor that is not chaotic, a small change will affect motion slightly while for a chaotic attractor the change will have dramatic results; this quality makes chaotic systems more flexible than would otherwise be possible.

The exponential sensitivity of Chaos is the very characteristic that makes it controllable. A slight displacement between the orbits of two identical chaotic systems will lead to a large difference; thus, a small error in the beginning can lead to an unavoidable separation between the behaviors of the two systems. This disadvantage is what Ditto et al. state as an asset of the chaotic behavior because with only a small perturbation we can control chaos. A chaotic system with unstable periodic motion contains corresponding trajectories in the phase space in a narrow space. These trajectories, which correspond to unstable periodic motions, can be moved by a small kick; it can help the system to jump from a periodic motion to another, among several.[88]

Another characteristic that helps to control Chaos is its deterministic nature. A chaotic system presents orbit complexity that Ott and Spano describe as the "many different kinds of motion that are possible on a chaotic attractor" and explain that "the attractors contain an infinite number of unstable periodic orbits."[89] This observation means that a chaotic system will have a periodic motion for a brief time and suddenly jump to another motion with a new period four times the previous one. At first glance, the change from one (unstable) periodic motion to another gives the impression of randomness; however, this is a deterministic phenomenon. The chaotic behavior is unstable and complex, but it has no relation with randomness and indeterminacy.[90]

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[87] Ott et al., "Controlling Chaos," 92.

[88] Ditto et al., "Techniques for the Control of Chaos," 199.

[89] Ott and Spano, "Controlling Chaos," 93.

[90] I explain the difference between random and non-random phenomena in Chapter I, section C.2, "Randomness, Chaos, and chaos."

B. CONTROLLING CHAOS WITH THE OGY METHOD

To implement the OGY method we need to use the Poincare sections. In Chapter II, I introduced the phase-space diagrams that contain the trajectories of chaotic systems. These trajectories for simple chaotic systems like the DDP contain all the essential information to predict the future behavior of the system. However, for more complex systems we need to use the Poincare sections that are one dimension smaller than the phase-space diagrams. Ditto et al. describe the Poincare section of a phase-space trajectory for period one (Figure 13) as:

> A more useful representation can be obtained by cutting through the phase space with a plane which intersects the circle in two places. The infinite number of points on the circle trajectory has been reduced to merely two. If we further confine ourselves to directed piercings of the plane, we are left with only a single point. Such a Poincare section reduces our information to a manageable level.[91]

According to this explanation, the Poincare section indicates the system's periodicity. For the chaotic motion that contains a large number of periodic motions, we have the formulation of the chaotic attractors. To implement the OGY method we need to know the attractor of the system and its reaction to a small perturbation.[92]

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91 Ditto et al., "Techniques for the Control of Chaos," 200.

92 Ditto et al., "Techniques for the Control of Chaos," 200.

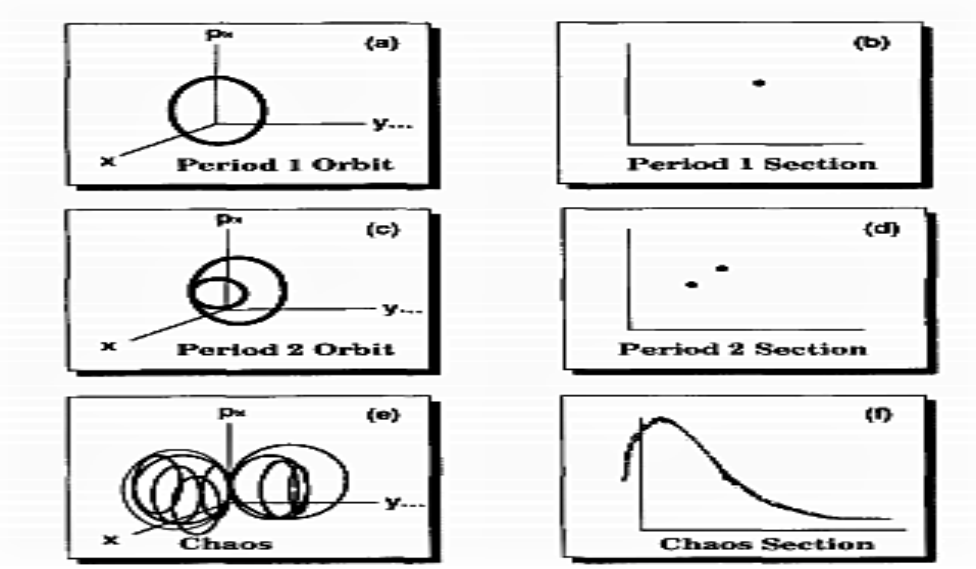

Figure 13. Phase-Space Trajectories for Period One, Two, and Chaotic Motion (left) and their Corresponding Poincare Sections (right).[93]

To implement the OGY method we first need to find a point of the attractor on the Poincare section that is unstable but periodic. In Figure 14, we see an *unstable fixed point* and its main characteristic is the close returns in the Poincare section. For experienced scientists this is an easy procedure, and for that reason, they have succeeded in controlling systems with a chaotic periodic motion of order up to 90; it also gives designers the opportunity to make flexible systems using Chaos control.[94]

The second step is to examine the shape of the attractor near the unstable fixed point. For this step, we observe the area next to the fixed point and how it moves. We analyze its motion in two directions; the stable and the unstable direction. We use eigenvectors for the motion of the current state of the system, which is called the *system state point*.[95] According to the departures or the approaches of the system state point

[93] Source: Ditto et al., "Techniques for the Control of Chaos," 200.

[94] Ibid., 201.

[95] An eigenvector is the vector that does not change direction if we apply a linear transformation on it, and the eigenvalue is the characteristic value of the eigenvector.

from the unstable fixed point, we have, respectively, the unstable or stable eigenvalue for the eigenvectors.[96]

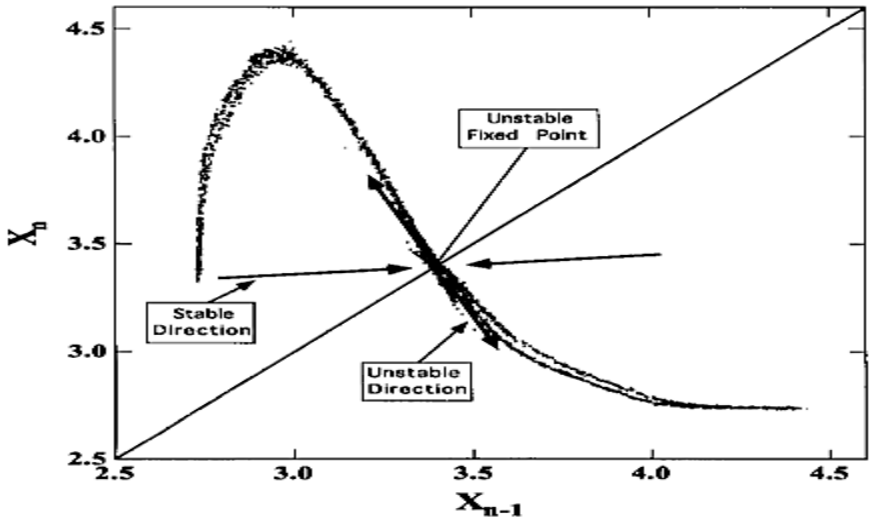

Figure 14. An Unstable Fixed Point on a Chaotic Attractor.[97]

The last step is to change slightly one of the system's parameters (Figure 15). We measure the fixed point for the several values of the parameter and we have the alteration of the attractor. The attractor's behavior in conjunction with the two steps integrates our knowledge of the chaotic system so as to control it. With the behavior of the fixed point to be known, we apply a small perturbation on the system, causing the system state point to move closer to the fixed point. This perturbation is applied once every period until the system state point reaches the fixed point, and this is what we call control of Chaos (Figure 16).

[96] Ditto et al., "Techniques for the Control of Chaos," 201–202.

[97] Source: Ditto et al., "Techniques for the Control of Chaos," 202.

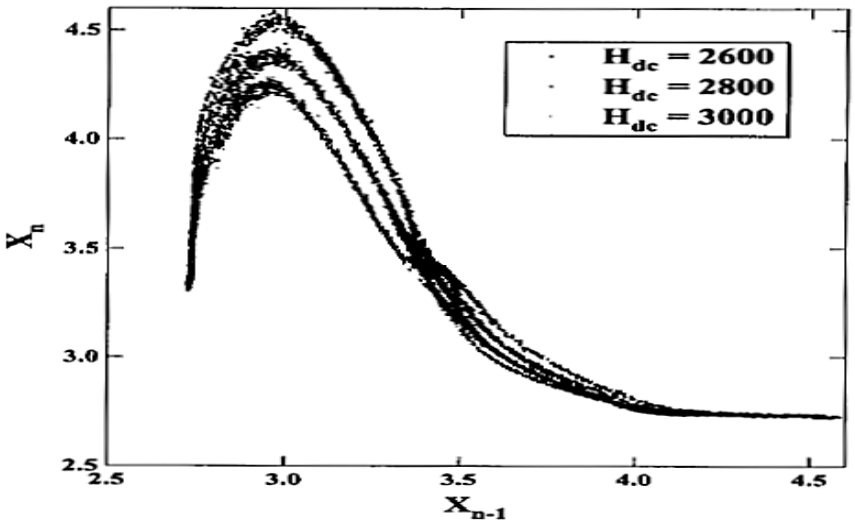

Figure 15. The Change of a Chaotic Attractor to a Change in a System Parameter H_{dc}.[98]

The formula that express this perturbation is

$$\delta p_n = C f_u^T \delta x_n ,$$

where δp_n is the value of the perturbation; in other words, it is the amount one needs to change the system's parameter to achieve control of the chaotic system. The value of δp_n depends on the distance of the system state point x_n from the fixed point x_F projected to the unstable direction f_u.[99] The constant C is arising from the previous measurements.

[98]Source: Ditto et al., "Techniques for the Control of Chaos," 201.

[99]Ibid., 204.

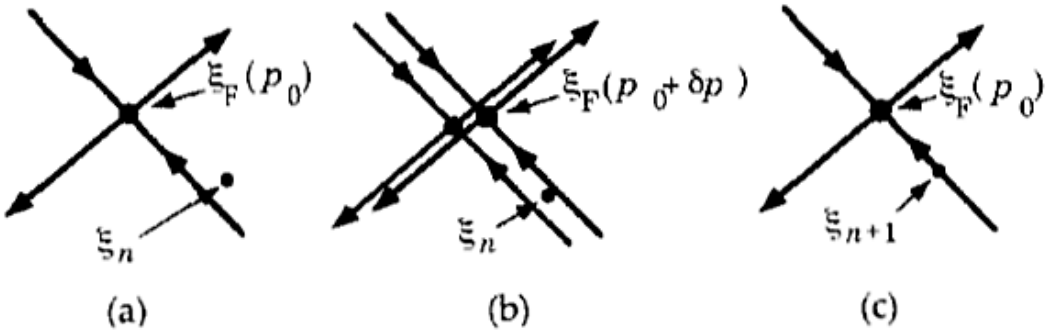

Figure 16. An Outline of the OGY Control Method in Three Steps. [100]

Ditto and Kenneth Showalter describe the OGY method as the stabilization of a ball (system state) on a saddle with the help of a perturbation (Figure 17).[101] One of the important advantages of this method is that the system maintains its chaotic nature, while other techniques remove the chaotic phenomena by suppressing Chaos.

[100] Source: Ditto et al., "Techniques for the Control of Chaos," 204.

[101] William L. Ditto and Kenneth Showalter, "Introduction: Control and Synchronization of Chaos," *AIP Publishing Chaos* 7 (1997): 509.

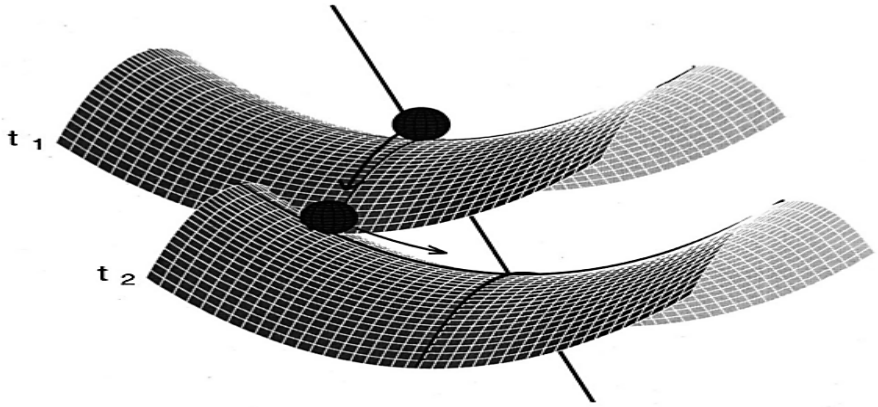

Figure 17. Stabilizing a State with One Stable Direction and One Unstable Direction by a Perturbation.[102]

C. APPLICATIONS OF CHAOS CONTROLLABILITY

Since the term Chaos was first introduced (1975) and the first attempt to control chaotic phenomena occurred (1990), several methods of control of the chaotic processes have been developed. The physicists Boris R. Andrievskii and Alexander L. Fradkov analyze the methods: *Open-Loop Control*, *Linear and Nonlinear Control*, *Adaptive Control*, *Linearization of the Poincare Map* (OGY method), *Time-Delayed Feedback* (Pyragas method), *Discrete Systems Control*, *Neural Network-Based Control*, and *Fuzzy Systems Control*; these are the most important from the dozens of their different versions. The scientists try to manage different chaotic phenomena such as identification of chaotic systems, controllability of chaos, chaos synthesis, synchronization of chaotic systems, tracking chaos, and chaos in control systems.[103]

All the aforementioned methods developed to manage chaotic phenomena have been successfully implemented. The first experimental control of chaos was achieved on a chaotically oscillating magnetoelastic ribbon.[104] Ditto, S. N. Rauseo, and Spano used

[102] Source: Ditto and Showalter, "Introduction: Control and Synchronization of Chaos," 509.

[103] Boris R. Andrievskii and Alexander L. Fradkov, "Methods and Applications," *Automation and Remote Control* 64, no. 5 (2003): 698–699.

[104] Magnetoelastic ribbon is a thin ribbon sensitive to electromagnetic forces.

the OGY method successfully to stabilize the unstable trajectories of period one and period two of this chaotic mechanical system.[105] With the OGY method the physicist Earle Hunt also stabilized the unstable periodic orbits of a driven diode resonator; the expansion of control to electronics was more successful as Professor Hunt achieved control of chaotic oscillations for high frequencies.[106]

By imitating and improving the OGY method, Rajarshi Roy et al. controlled a multimode, autonomously chaotic solid-state laser system, and they named their control *the technique of occasional proportional feedback*.[107] They argued that their technique would be "widely applicable to autonomous, higher-dimensional chaotic systems, including globally coupled arrays of nonlinear oscillators."[108] Valery Petrov et al. also applied a method similar to OGY to control the chaotic behavior of an oscillatory chemical system, the Belousov-Zhabotinsky reaction.[109] They were able to track period one after the first period doubling bifurcation and to implement their control algorithm later.[110]

From all these examples, we can conclude that the OGY method and all the other techniques have several applications. We have measureable results and applications of the control of Chaos for mechanical systems, electronics, lasers, chemical systems, and biology (heart and brain tissue). With all these cases, natural scientists have proved that Chaos exists, and they have developed methods to control and exploit the chaotic phenomena. Chaos Theory works for the natural sciences!

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[105] W. L. Ditto, S. N. Rauseo, and M. L. Spano, "Experimental Control of Chaos," *Physical Review Letters* 65, no. 26 (1990): 3211–3215.

[106] Raymond E. Hunt, "Stabilizing High-Period Orbits in a Chaotic System: The Diode Resonator," *Physical Review Letters* 67, no. 15 (1991): 1953–1955.

[107] Rajarshi Roy, T. W. Murphy, Jr., T. D. Maier, Z. Gills and E. R. Hunt, "Dynamical Control of a Chaotic Laser: Experimental Stabilization of a Globally Coupled System," *Physical Review Letters* 68, no. 9 (1991): 1259–1262.

[108] Ibid.

[109] The Belousov-Zhabotinsky reaction is a chemical reaction that produces a nonlinear chemical oscillator that evolves chaotically.

[110] Valery Petrov, Vlmos Gaspar, Jonathan Masere, and Kenneth Showalter, "Controlling Chaos in the Belousov-Zhabotinsky Reaction," *Nature* 361 (1993): 240–243.

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IV. CONVENTIONAL THEORIES FOR INTERNATIONAL RELATIONS ANALYSIS

The German philosopher Carl Schmitt stated that "all genuine political theories presuppose man to be evil."[111] Schmitt indicated that crises and conflicts are the enemies of humanity. These dynamic evils appear in various forms and have the ability to influence politics, economies, and ethics, ending up in an undeclared war against humanity. The modern political challenge is to predict those crises, using or modifying existing political theories. Over the years, political scientists have made several efforts to determine the most effective and equitable political theory for humanity. Achieving a balance between effectiveness and ethics in political theories is challenging. From this perspective, defining the perfect political theory is a matter of considerable controversy; however, the modern world seems to be polarized between two major political theories: Realism and Liberalism. This indicates the importance of the political scientists' efforts to utilize natural science models and to apply those theories in the real world. In this chapter, I examine the meaning of those theories and their historical orientation. Then, I compare and contrast them.

A. REALISM

The first traces of realistic ideology are found in ancient Greece, where Plato used this definition to define universal and abstract objects. Around the same time, Thucydides reveals in his book, *The History of the Peloponnesian War*, the initial values of realism, while he describes a conflict between two Greek ancient cities. Historically, ethics have proven inadequate to constrain the human desire to gain power. Machiavellianism emerged as a new form of realism during the Middle Ages. According to Machiavellianism, morality and politics are incompatible, and states should use all their means to achieve specific benefits, even if they are unethical.[112] In the 18th century, Carl

[111] Heinrich Meier, *The Lesson of Carl Schmitt: Four Chapters on the Distinction between Political Theology and Political Philosophy* (Chicago: University of Chicago, 1998), 80.

[112] Julian Korab-Karpowicz, *On the History of Political Philosophy: Great Political Thinkers from Thucydides to Locke* (London: Routledge, 2011), 115–140.

von Clausewitz applied realism to war activities, as the remedy that leads warriors to victory.[113] Sequentially, by the early years of the 19th century realism expanded to embrace art (Figure 18), science, aesthetics, politics, economics, and other major arenas of our society.

Figure 18. "A Burial at Ornans" (1849–50), Gustave Courbet, the Realist Movement.[114]

Hans J. Morgenthau was the first to introduce realism in international relations during the 20th century. His theory revealed the human desire for domination prevails over the desire for cooperation, and this explains human aggressiveness; following their human nature, politicians crave power in order to promote their interests.[115] Morgenthau separates ethics from politics, since he recommends that politicians should sacrifice

[113] Clausewitz developed the idea that "in a war, the ends justify the means."

[114] Source: Ditto and Showalter, "Realism," The Art Story: Modern Art Insight, accessed December 14, 2016, http://www.theartstory.org/movement-realism.htm.

[115] Hans Morgenthau, *Politics among Nations: The Struggle for Power and Peace* (Boston: McGraw-Hill, 1993), 5–9.

ethics on behalf of a successful political choice. Power should guide politicians instead of morality, and their choices should be autonomous and decisive.[116]

Realism significantly affected the IR field. It is common for theorists to support and compare their arguments in IR with three images, which the realist Kenneth Waltz introduced in 1954 with his book *Man, the State, and War*. With the first image in IR he refers to *human nature*, with the second image to *the state*, and with the third image to *the international system.*[117] Waltz, states that "to build a theory of international relations on accidents of geography and history is dangerous."[118] With this quote he implies that theories should take into account several parameters, as the states are not perfect and their behavior is not predefined. For several years realism, as it was expressed by Waltz, was a compelling theory for scholars, because it was a multi-criteria theory that contained fewer constraints to explain international relations.

Realism's key concept is the interest of power, and realists conceive the international system as anarchy. Morgenthau maintains that states should act in order to seek more power as that can provide them with security; the distribution of power determines the international order.[119] The drive for power and security, however, cannot create a conducive environment for cooperation among nations and often states prefer competition rather than cooperation.[120] In the same vein, Thucydides maintains that power is the most important factor as international affairs do not contain "romantic elements" or moral dilemmas. By saying "what made war inevitable was the growth of Athenian power and the fear which this caused in Sparta," Thucydides imparts to his analysis the element of the third image as the first and second images are not enough to justify why Sparta declared the Peloponnesian war.[121]

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116 Ibid.

117 Kenneth Waltz, *Man, the State, and War: A Theoretical Analysis* (New York: Columbia University Press, 1959), 216–223.

118 Ibid., 107.

119 Morgenthau, *Politics among Nations*, 5–9.

120 Ibid.

121 Thucydides, *History of the Peloponnesian War*, Rex Warner, trans. (New York: Penguin Books, 1954), 48–49.

While classical realism contends that human nature leads states in how to shape their policy, Waltz argues that the first image is not the only factor, as the international system and the internal situation of a state can affect the behavior of the state. He stresses that emphasizing one image can distort the other two and proposes a holistic approach where the analysts should examine how the first and second image are interrelated with the third.[122] What makes Waltz's theory unique is his realization that we should escape from "the belief that international-political outcomes are determined" and his argument that states' efforts for security can leave them less secure.[123]

Realism declares international relations is a system of anarchy, since there is no rule of enforcement established by a superior authority.[124] Cooperation in international relations is a game that the most powerful nations establish in order to increase their power among the other developed states without risking a war.[125] These powerful players enforce the rules for smaller nations, which are seeking their chance to increase their own strength through adaptation in the new geopolitical and economic environment. On the other hand, the strongest individuals and the firms, in each nation, suppress those efforts for cooperation, while they promote their international interests.[126]

According to realism, human nature is evil, greedy, and competitive. People are always seeking power, and they tend to fight each other in order to promote their interests. This ideology assumes a powerful state's need to accumulate power in order to secure its interests and force other states to comply with its priorities.[127] In other words, realism supports the idea of having powerful and effective armed forces, as a means of intimidation against other countries. Nations and states are the primary elements of realism, and in favor of their security preemptive war, actions can be excused.

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[122] Waltz, *Man, the State, and War*, 231–235.

[123] Kenneth Waltz, *Theory of International Politics* (New York: McGraw-Hill, 1979), 77–78.

[124] Robert Axelrod and Robert O. Keohane, *Achieving Cooperation under Anarchy: Strategies and Institutions* (Cambridge: Cambridge University Press, 1985), 226.

[125] Morgenthau, *Politics among Nations*, 5–9.

[126] Axelrod and Keohane, *Achieving Cooperation under Anarchy,* 230.

[127] Heinrich Meier, *The Lesson of Carl Schmitt* (Chicago: University of Chicago), 1–26.

Classic Realism, Neorealism, and Neoclassical Realism are some of the several forms of realism in politics and international relations that relate to human nature. Realism supports the rights of the most powerful and defines that the ends justify the means. From this point of view, conflicts in the arena of the international relations are inevitable, while states and governors seek power.[128] Classic realism supports that this competition rises from human nature itself and the craving for power, while Neorealism supports that this battle is rooted in the system of anarchy among the most powerful states. Lastly, neoclassical realists advocate for both sides, while they add that we should take into account some domestic parameters of states themselves. Neoclassical realism also supports the notion that human nature in conjunction with the distribution of power among states shapes the foreign policy of a state.[129]

Another major aspect of realism is the international distribution of power that should be with limited polarity.[130] Realism asserts that the multipolar state-system, which is constructed from more than three poles of power, is not a stable system.[131] On the other hand, a unipolar system would also be inevitable, because smaller or less powerful states are likely to cooperate in order to change the power balance on behalf of their interest. From this perspective two or three states, despite their nature to dominate over the others, can cooperate to establish a bipolar power system in this world.[132] As in the case of the Cold War, the relationship between the U.S. and the U.S.S.R. seemed more complicit than cooperative. It also seems to have evolved. The governments of these nations did not purposely meet and sign a specific agreement to become the only two superpowers. This power system would be more stable and would support the powerful states' thirst for domination, more effectively.

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[128] Annette Freyberg, Inan E. Harrison, and Patrick James, *Rethinking Realism in International Relations: Between Tradition and Innovation* (Baltimore, MD: The Johns Hopkins University Press, 2009), 21–26.

[129] Ibid., 1–6.

[130] With the term polarity, Chantal Mouffe means the number of states in the globe that are able to control and dominate the others.

[131] Chantal Mouffe, "Democracy in a Multipolar World," *Millennium: Journal of International Studies*, vol. 37, no. 3 (2009): 549–561.

[132] Waltz, *Man, the State, and War: A Theoretical Analysis*, 80–86.

Realists argue that states are the most powerful element in the world scene, while they claim that the influence of the non-governmental institutions or individuals is of minor importance. Realists support the notion that there are no universal authorities that can control and manipulate the governors in a specific way in order to promote their states' interests.[133] The realists do not consider the U.N., NATO, or the international court in The Hague capable of exerting any influence in world affairs. On the other hand, powerful states are aggressive, and they invest in their safety; their priority is to establish external and internal security. Once they achieve the desired security through a strong presence of armed forces, then they focus on amassing power and control over other states.[134]

For realists, the economy is also of utmost importance as it is the primary means to gather and conserve power. Colonialism is the key element of the realist economy and the ultimate tool for the realist governments to gain power and promote their interests.[135] Neo-colonialism, mercantilism, and ethno-nationalism are later forms of colonialism, and historically they were the keystone of realist economic system. The principles of free trade and competitiveness dominate in all these systems and they are the precursors of globalization.[136] Each of these constructs exhibits characteristics of free trade, capital markets, and open economies. Self-serving interests and progress are the basic focus areas for a state, according to realist economic ideology.

B. LIBERALISM

Liberalism is the antithesis of realism. Maurice Cranston states that "a liberal is a man who believes in liberty."[137] Liberalism is rooted in the English revolution of 1688,

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[133] Chris Brown, "*Understanding International Relations,*" 3rd ed. (New York: Palgrave McMillan, 2005), 63–65.

[134] Ibid., 91–93.

[135] Jonathan Kirshner, "Realist Political Economy: Traditions Themes and Contemporary Challenges," in *Routledge Handbook of International Political Economy*, ed. Mark Blyth (New York: Routledge, 2009), 37–43.

[136] Ibid., 40–41.

[137] Maurice Cranston, 'Liberalism,' in *The Encyclopedia of Philosophy*, ed. Paul Edwards (New York, New York Press, 1967), 458–461.

while John Locke and Montesquieu are considered to be the fathers of this movement.[138] In 1776 the declared independence of the United States of America reinforced liberal principles since it was promoting equality among people. The French Revolution, some years later, was based on the principles of equality and liberty. The Enlightenment in Europe was also a movement in the direction of liberalism, based on the ideas of fraternity, liberty, and equality among people and nations.

During the 18th century, the Scottish philosopher, Adam Smith was a major pioneer of liberalism, known as the founder of the free-market economic model. During the next century a similar movement, known as Social Liberty, raised the individual's political rights and the society's thirst to control its governors through constitutional limitations.[139] Later, President Franklin D. Roosevelt's social liberal campaign promoted the liberal economic model in the United States and introduced a new era in the global economy.

Since liberalism evolved into a very popular ideology during the last few centuries, it is important to define its meaning in politics and international relations. Liberalism claims that man should be absolutely free within a state to perform any kind of economic, political, social, and religious action.[140] The decisions of humans or states should be a product of free will and independent of any kind of external power. With regard to the aforementioned world-power polarity, liberalism promotes the multipolar model of cooperation among different states. According to liberalists this model is more equitable and moral, protecting the human rights of the individual within each state.[141]

Liberalism in contrast with realism considers cooperation—not competition—as the key element for international relations and argues that institutions have a vital role for that. Liberal thought, as does realism, admits that the nature of the international system is

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138 Richard Aaron, *John Locke* (Oxford: Oxford University Press, 1972), 1–5.

139 Robert Nozick, *Anarchy, State, and Utopia* (Oxford: Blackwell, 2013), 18–21.

140 John Locke, *The Second Treatise of Government in Two Treatises of Government*, ed. Peter Laslett (Cambridge: Cambridge University Press, 1960), 283–285.

141 Chantal Mouffe, "Democracy in a Multipolar World," *Millennium*, vol. 37, no. 3 (2009): 549–561.

anarchic; however, liberalists argue that "democracies almost never fight each other."[142] For liberalists, units of analysis are not the states but the relations between the international actors.[143] These relations can be determined by friendship and cooperation or by competition and conflict, and the former can prevail over the latter if the states are improved democracies with strong relations—economic, political, and cultural—among them. Bruce Russett argues that some wars—World War I and II—cannot validate that democracies fight each other because he disputes that the engaged states were democracies.[144]

In liberalism any kind of limitations with regard to the individuals' actions should be totally justified through a flexible legal framework, which has priority to conserve the free will of civil society in each state. On the other hand, the state should be the perfect shelter for the rise of new ideas, so progress can occur through the efforts of individuals to improve their lives. According to liberalism, development is in harmony with the rights and the liberty of the civil society. From this point of view, governmental organization should focus on the security of the individual as the only way to evolution.

Neo-liberals extend the notion of cooperation while they admit that states have many difficulties to overcome.[145] They argue that "the more future payoffs are valued relative to current payoffs, the less the incentive to defect today"; they conclude that states should develop strategies to overcome their differences and to base their actions on reciprocity and shared beliefs.[146] Robert Axelrod and Robert O. Keohane stress that all of

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[142] Bruce Russet, *Grasping the Democratic Peace: Principles for a Post- Cold War World* (Princeton, NJ: Princeton University Press, 1993), 58.

[143] Ibid., 74–81.

[144] Bruce Russet uses Huntington's definition of a democracy, which describes the criteria as: "a twentieth-century political system as democratic to the extent that its most powerful collective decision makers are selected through fair, honest, and periodic elections in which candidates freely compete for votes and in which virtually all the adult population is eligible to vote." Russet, *Grasping the Democratic Peace*, 14.

[145] Neo-liberalism extends the notion of liberalism by emphasizing the value of free market competition.

[146] Robert Axelrod and Robert O. Keohane, "Achieving Cooperation under Anarchy: Strategies and Institutions," in *Neorealism and Neoliberalism: The Contemporary Debate,* ed. David A. Baldwin (New York: Columbia University Press, 1993), 226–232.

these challenges are very difficult but not impossible. They explain that cooperation in world politics cannot be achieved through unilateral benefits (liberal republicanism) but with a developed sense of reciprocity which "requires the ability to recognize and retaliate against defection."[147]

In economic terms liberalism is based on two basic values. The first one arises from the French phrase *Laissez-faire*, which means let them do it.[148] In other words, the economy of a state should have its own rules and should balance according to the offer-demand law. Government intervention should be avoided in order to promote the second value of economic liberalism: the free market.[149] The free-market model was introduced by Adam Smith. According to this model the state's role should be distinguished from the economic society. Companies should perform according to their interests in order to increase their profits, and states should support this effort by providing a liberal environment.

Liberalists introduce new actors into the international political scene. Such actors include multinational companies and universal non-governmental organizations. National borders and external security, in terms of powerful armed forces, are under dispute according this theory.[150] The primary value is cooperation between nations and states, so that companies can change and guide the global market. This cooperation should replace the need for border security and armed forces.[151] In other words, liberalism is the keystone of capitalism, which serves the rights of wealth.

C. COMPARISON OF LIBERALISM AND REALISM

By comparing the founders, the thinkers, and the implementers of the two theories, we can conclude that realism and liberalism explain differently how the

147 Ibid., 249–250.

148 Gilbert Faccarello, *The Foundations of Laissez-Faire* (New York: Routledge, 1999), 34–58.

149 Ibid.

150 Eric B. Shiraev, *International Relations* (New York: Oxford University Press, 2014), 78.

151 Loren E. Lomasky, *Persons, Rights, and the Moral Community* (New York: Oxford University Press, 1987), 84–115.

international system works and how it will achieve peace and stability. The only common characteristic for both theories is that the nature of the international system is anarchic; however, for the realists this anarchic system will exist forever while for the liberalists the anarchy will end if the states act collectively. By analyzing the two theories through the IR lens, we can summarize the main differences in five terms: the state, the aim of the states, the relation between the states, the peace, and the institutions; all these terms are defined differently by each theory (Figure 19).

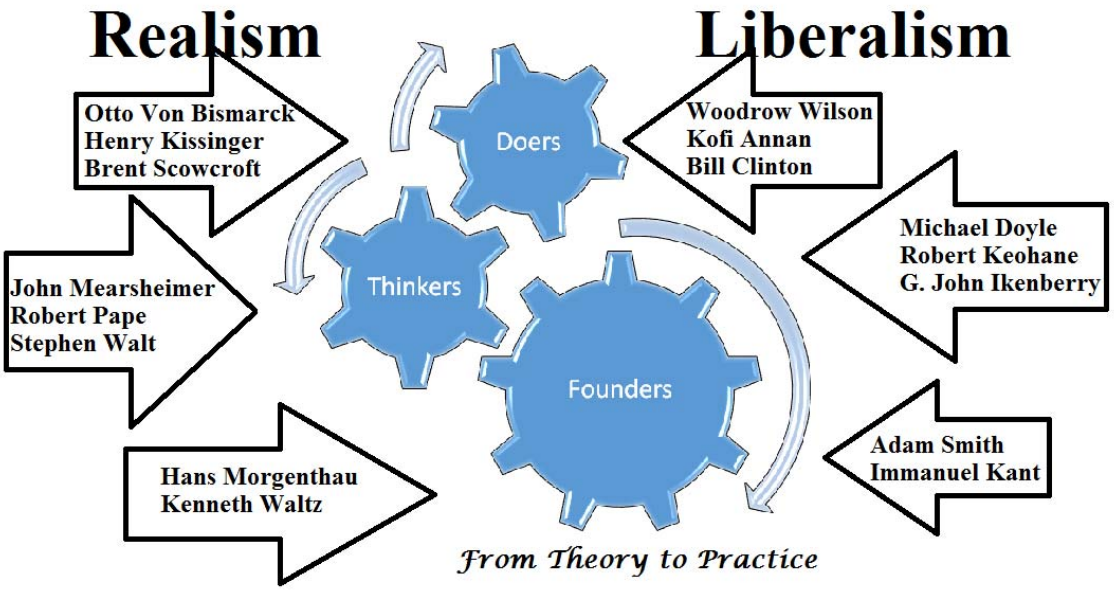

Figure 19. Realism versus Liberalism, from Theory to Practice.[152]

The different ways that realism and liberalism define the state explain how these theories describe the roles and the aims of the states. For realists the state is independent and autonomous, while for liberalists the state acts according to the political, economic, and social actions of other states. Liberalism asserts that states should follow strategies that respect the international laws and norms, and the national interest is not well defined

[152] Adapted from "Realism versus Liberalism," *Westphalian Post*, https://westphalianpost.wordpress.com/machtpolitik/.

as states should act collectively. [153] On the other hand, for realists the states have internal and external sovereignty and their strategies have to serve the national interest in order to increase their power.[154] The role of the state in the international system is very significant for realists, so other actors, like organizations, institutions, international companies, religious groups, are less important. By contrast, for liberalists these actors diminish the role of states and formulate international relations.[155]

The aims of states and the relations that they have are determined differently according to each theory. According to realism, states continuously aim to increase their power and their influence so as to dominate in the international system and to secure their independence. This dominance has only economic parameters for liberalists, as they support the notion that states want to maximize their economic development.[156] The relations between states are competitive for realists, and the nature of this competition is economic and geopolitical (armed forces, diplomacy, influence). Liberalists accept only the economic aspect of this competition, and they separate the power of a state and its wealth.

The terms of peace and institutions have different definitions for realists and liberalists. Realists assert that the balance of power is the mean for international stability and peace; while liberalists argue that the institutions, the democratic regimes, and the free markets are essential elements of peace (Figure 20).[157] According to political realism, international institutions and security coalitions serve the national interests of the superpowers, and they act according to the directions of the superpowers. In sharp

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[153] Russett, *Grasping the Democratic Peace,* 74–81.

[154] Annette Freyberg, Inan E. Harrison, and Patrick James, *Rethinking Realism in International Relations: Between Tradition and Innovation* (Baltimore, MD: The Johns Hopkins University Press, 2009), 21–26.

[155] Ibid.

[156] Jonathan Kirshner, "Realist Political Economy: Traditions Themes and Contemporary Challenges," in *Routledge Handbook of International Political Economy*, ed. Mark Blyth (New York: Routledge , 2009), 37–43.

[157] Russett, *Grasping the Democratic Peace,* 58.

contrast, the liberalists support the notion that the stability of the international system depends on the independent function of these institutions.[158]

Figure 20. Realism versus Liberalism in International Relations.

D. CONCLUSIONS

If we consider that the role of IR theories is to help policy makers to make better decisions, this chapter shows that there will be a wide spectrum of choices for them. Liberalism and realism are two theories that have different answers for the same questions. The way that each theory explains the political phenomena around the world can be controversial and sometimes confusing for decision makers. The origins of the instability have different roots for each theory and the paths to the peace are in different directions.

[158] Freyberg et al., *Rethinking Realism in International Relations*, 21–26.

The founders of these theories tried to explain the functions of the international system with abstract concepts, and the scholars tried to develop them; however, we may have to reconsider or to improve some elements of these theories, as they explain very simply, complex international relations. To create order among the interactions of different nations, these theories use the "if…then" model, which is the method for experiments in the natural sciences. The above comparison give us the hope that the models of physics can help with this "if…then" procedure.

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V. APPLYING SAPERSTEIN'S CHAOTIC MODEL TO SELECTED INTERNATIONAL RELATIONS THEORIES

The famous mathematician John von Neumann once observed that "the sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena."[159] With this statement Neumann expressed the human ability to describe the real world with models, and the efforts we do to predict the future by observing current phenomena. For that reason the physicist Alvin Saperstein has asserted that "applications of some of the mathematical modeling methods of the physical sciences to the social sciences can only strengthen the latter."[160] Saperstein's chaotic model is able to give reasonable answers to several IR theories and hence promises that CT can be a useful tool for IR analysts.

In this chapter, I present two conventional IR theories that Saperstein applies his CT model, and I describe Saperstein's model. Second, I review the results that the model gives to the questions: "Which is more war prone—a bipolar or a tripolar world?" "Are democracies more or less prone to war?" Finally, I compare these answers from the standpoint of conventional theories.

A. TWO INTERNATIONAL RELATIONS THEORIES THAT SAPERSTEIN USES TO TEST CHAOS.

Alvin Saperstein uses two theories to test his chaotic model. The first is Bruce Russett's IR theory—the states with democratic regimes are not fighting each other—and the second is Mearsheimer's IR theory—that a bipolar world is more stable than a multipolar structure.

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[159] John von Neumann. AZQuotes.com, Wind and Fly LTD, accessed December 13, 2016, http://www.azquotes.com/author/10753-John_von_Neumann.

[160] Alvin M Saperstein, *Dynamical Modeling of the Onset of War* (Singapore: World Scientific Publishing, 1999), 5.

On April 2, 1917, Woodrow Wilson, in his war message to the Congress states that "self-governed nations do not fill their neighbor states with spies or set the course of intrigue to bring about some critical posture of affairs which will give them an opportunity to strike and make conquest."[161] This statement is in the same vein with Immanuel Kant who argued that the states with *republican constitutions* have perpetual peace.[162] Such arguments are the base that Bruce Russett uses to develop the idea that democracies do not fight each other; he supports the vision of peace among democratically governed states; Saperstein uses this theory and with his model tests its accuracy.

Bruce Russett summarizes some hypotheses that explain the causal mechanism of this theory and explains that the reasons for peace are rooted in democracy. First hypothesis is that transnational and international institutions make peace. For Russett, the international organizations and institutions aim to protect common interests between the member states. The European Union is an example of such institutions that protect—previously hostile—member states so they do not to fight one another.[163] Russett's second hypothesis is that alliances make peace; the allies choose each other, and that makes the war unlikely.[164] Third is that wealth makes peace, and he argues that democracies are often wealthy. The wealthy states support the political stability and the costs of the war are more than the benefits. The transnational interests of trade and investment are of utmost importance for the wealthy states.[165] The last hypothesis for Russett is that political stability, which is a characteristic of the democratic states, helps the states avoid conflicts. The unstable governments are prone to war with adversary states that face domestic political problems.[166]

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[161] Russett, *Grasping the Democratic Peace,* 3.

[162] Russett, *Grasping the Democratic Peace,* 4.

[163] Ibid., 25.

[164] Ibid., 27.

[165] Ibid., 28.

[166] Ibid., 29.

Bruce Russett's democratic peace theory is not expressed exactly through the Saperstein's question "are democracies more or less prone to war?" however it is essential to format the question in this way. The democratic theory expresses that democracies are as bellicose as non-democracies, but they tend to not fight each other as opposed to fighting non-democratic states. While the right question for Saperstein should have been "why do democracies not fight each other?" he made a slight change to the question so as to be compatible with the model. The models have the ability to answer to questions by choosing from a variety of responses that the modeler has predefined; however, it is impossible to answer to why-questions as the responses for such questions are descriptive and the models do not give descriptive answers.

The second theory that Saperstein tests is John Mearsheimer's assessment for the stability of the international system. Mearsheimer argues that a bipolar structure benefits the stability and he explains that the end of the Cold War could destabilize the whole world.[167] He argues that "the prospects for major crises and war in Europe are likely to increase markedly if the Cold War ends…this pessimistic conclusion rests on the argument that the distribution and character of military power are the root causes of war and peace."[168]

Mearsheimer argues that the bipolar system is stable and he uses the example of the Cold War. He describes that there are three factors that provided stability during the Cold War era: "the bipolar distribution of military power on the Continent; the rough military equality between the two states comprising the two poles in Europe, the United States and the Soviet Union; and the fact that each superpower was armed with a large nuclear arsenal."[169] For the above reasons Mearsheimer supports that a bipolar system is more peaceful as there are fewer conflict dyads, fewer imbalances of power, and fewer miscalculations of relative power and of opponents' resolve.[170]

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[167] John J. Mearsheimer, "Back to the future: Instability in Europe After the Cold War," International Security 15, Vol. 1, 1990, 5.

[168] Ibid., 6.

[169] Ibid., 7.

[170] Ibid., 14.

B. QUALITATIVE DESCRIPTION OF SAPERSTEIN'S CHAOTIC MODELS

A mathematically sound and effective method, within mathematical limits, is the proposal of Saperstein to use Chaos Theory on sub-systems of the real world to predict the unpredictable.[171] He supports his arguments with Clausewitz's idea that war is a chaotic process, and we may predict its outbreak but not its outcome.[172] Saperstein makes a separation between hard and soft Chaos in IR. A system that reacts in a specific environment and receives specific inputs will provide outputs. If the fluctuation of these outputs is small compared to the extent of the system, though large with respect to the inputs, we have the case of soft Chaos. In hard Chaos, the fluctuations dominate the entire system, and we have a totally unpredictable situation.[173] The prediction of hard Chaos in a model is a warning to policymakers, because the unpredictability of hard Chaos represents crisis, instability, and war in the international system. This construction is similar to physicists' approaches to Chaos. Physicists control chaotic consequences with small "kicks"; social scientists will respond to indications of chaotic consequences with warnings to change policy.

As an example for his models Saperstein gives convincing answers to different questions: "Are democracies more or less prone to war?", "Is a bipolar or tripolar World more stable?"[174] He proves that an approach to analyzing IR with the help of CT is possible according to his assumptions. With the help of simple mathematical models and equations he proves that democratic nations are more stable than autocracies and that a tripolar world is less stable.[175] Saperstein uses different ranges of parameters, different algebraic forms, and he checks which inputs lead to stable solutions and which lead to crisis or unstable situations. Saperstein's approach is very rigorous mathematically, and

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[171] Alvin M. Saperstein "The Prediction of Unpredictability: Applications of the New Paradigm of Chaos in Dynamical Systems to the Old Problem of the Stability of a System of Hostile Nations," in *Chaos Theory in the Social Sciences Foundations and Applications*, ed. Douglas Kiel and Eu el Elliott (Ann Arbor: The University of Michigan Press, 1996), 149.

[172] Ibid., 146–147.

[173] Ibid., 145.

[174] Saperstein "The Prediction of Unpredictability," 153–162.

[175] Ibid.

for that reason, an extensive mathematical analysis is beyond of the scope of this thesis.[176]

Saperstein argues that such models give us the hope of creating theoretical models that can correctly predict the unstable situations of real-world systems.[177] We can make models that do not violate mathematical notions and that are inspired by CT. For example, the stability of a mathematical model can help social scientists to understand the interaction between states. It can also work properly for an isolated system, and it can predict qualitative characteristics, such as the proneness to war.

As physicists have the pendulum to test Chaos Theory, social scientists can use history. If a model that relies on this theory works properly, it should work for the past events; we have to check whether we can use such models to identify periods of history marked by instability. In the same vein with Saperstein, Dimitrios Dendrinos stresses that social sciences should not persist in learning only from static, sharp, or stable dynamical models; Chaos Theory and its insights should be applied.[178] Analysts of IR issues will never have adequate accurate measurements but decision makers have to act, so a theoretical model to work for real world systems is a good option.

Saperstein uses the terms linearity and non-linearity and the terms stability and instability to conduct the experiment and reach his conclusions. He explains that models that are linear, are far from reality because they need a huge amount of data to give realistic predictions; the more data the model needs, the more useless it is.[179] However, the use of a chaotic model is closer to reality, as after some critical values in the phase-state diagrams (like the critical values of DDP in Chapter II) we have exponential growth. Saperstein argues that "given a specified non-linear theory, with its inherent possibility of producing bifurcations, Liapunov exponents [explained earlier in this thesis] can be

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[176] The entire set of equations and chaotic models that Saperstein uses are not analyzed.

[177] Ibid., 162–163.

[178] Dimitrios S. Dendrinos, "Cities as Spatial Chaotic Attractors," in *Chaos Theory in the Social Sciences Foundations and Applications*, ed. Douglas Kiel and Euel Elliott (Ann Arbor: The University of Michigan Press, 1996), 242.

[179] Saperstein, *Dynamical Modeling of the Onset of War*, 89.

calculated for 'typical' points."[180] For negative values of Lyapunov exponents we have stability and peace, while for positive values there is sensitivity to initial conditions so Chaos is present. Both predictions are useful for policy makers as they are informed about which actions can cause unstable situations, and Saperstein comments that "it is important to the policy maker to know whether the world he confronts is, or will be, stable or unstable in response to his anticipated changes."[181]

C. IMPLEMENTATION OF SAPERSTEIN'S CHAOTIC MODELS ON INTERNATIONAL RELATIONS ISSUES

Saperstein uses a chaotic model to analyze two different IR issues. The first issue is: "Which is more war prone—a bipolar or a tripolar world?" and the second is: "Are democracies more or less prone to war?" In order to answer the first question, Saperstein creates two different systems of equations. The first system is for a bipolar world, and it has the following two equations.

$$X_{N+1} = 4aY_N(1-Y_N)$$

$$Y_{N+1} = 4bX_N(1-X_N)$$

With the X_{N+1} and the Y_{N+1} we have the rate of the "devotion" that a nation shows in year N+1 in an arms race. To calculate this devotion, we measure the expenditures of a nation for military weapons and equipment; we also include the cost for the infrastructure that the nation supported to calculate the ratio of the total arms procurement to the gross national product. The rate X_{N+1} of the first state is proportional to the rate Y_N of the second state, because we assume that a state will spend proportionately on arms what its enemy spent the previous year. The same assumption is made for the second state, so the variable Y_{N+1} is proportional to the X_N. The α and b parameters are related to the Lyapunov exponent λ, and Saperstein calculates the region of stability as a function of these parameters (Figure 21).[182]

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[180] Ibid.

[181] Saperstein, *Dynamical Modeling of the Onset of War*, 90.

[182] Source: Saperstein, "Stability Plot for a Bi-polar Competitive System," 101.

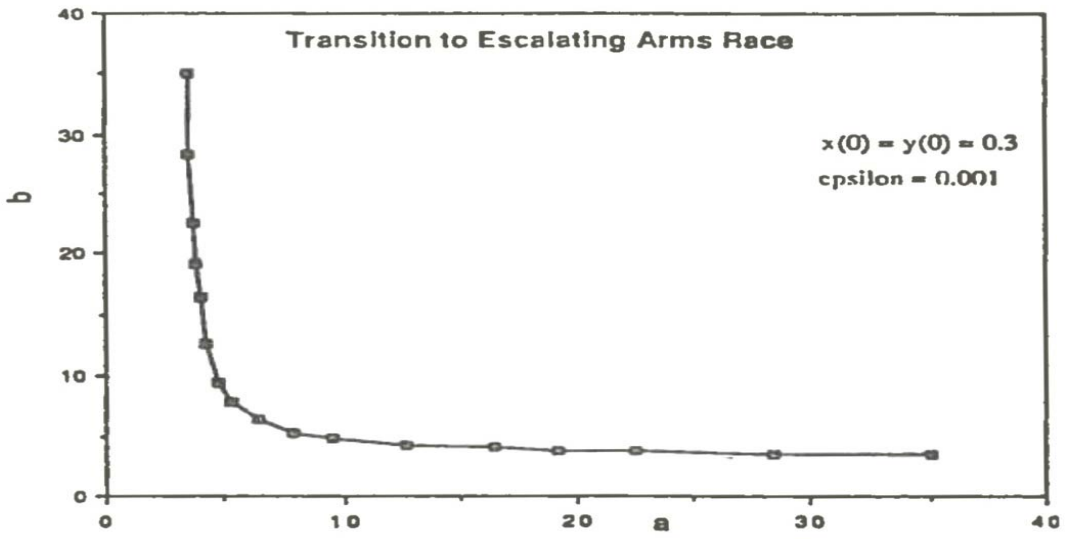

Figure 21. Stability Plot for a Bi-polar Competitive System.[183]

For the curve shown in Figure 21, Saperstein argues that:

> The resultant curve represents the critical relation between α and b in the unit square of the a-b plane. The region above the curve, in which the two Lyapunov coefficients $A(X_O)$ and $A(Y_o)$ are positive, is the model's chaotic region. Thus the square region in parameter space (0<a, b<1) is divided into crisis-stable and crisis-unstable regions for the system parameters α and b.

As this curve is not enough to provide complete information to answer our question, Saperstein extends the model to three nations and our new system is:

$$X_{N+1} = 4aY_N(1-Y_N) + 4\varepsilon Z_N(1-Z_N)$$

$$Y_{N+1} = 4bX_N(1-X_N) + 4\varepsilon cZ_N(1-Z_N)$$

$$Z_{N+1} = 4\varepsilon[X_N(1-X_N) + c\mathrm{Y}_\mathrm{N}(1-\mathrm{Y}_N)]$$

For $\varepsilon = 1$ the system will be the previous one (the two nations' system). Now, we can come to conclusions because for large values of ε we have more coupling between the variables X, Y, and Z. At this point Saperstein states that "with numerical computations of the Lyapunov coefficients, the stability region decreases in area as epsilon increases,

[183]Source: Saperstein, "Stability Plot for a Bi-polar Competitive System," 105.

i.e., as the third nation becomes more significant in the world system."[184] With that observation we have that the international system is more unstable when the third nation has the role of a superpower. That is, a tripolar world is less stable (Figure 22).

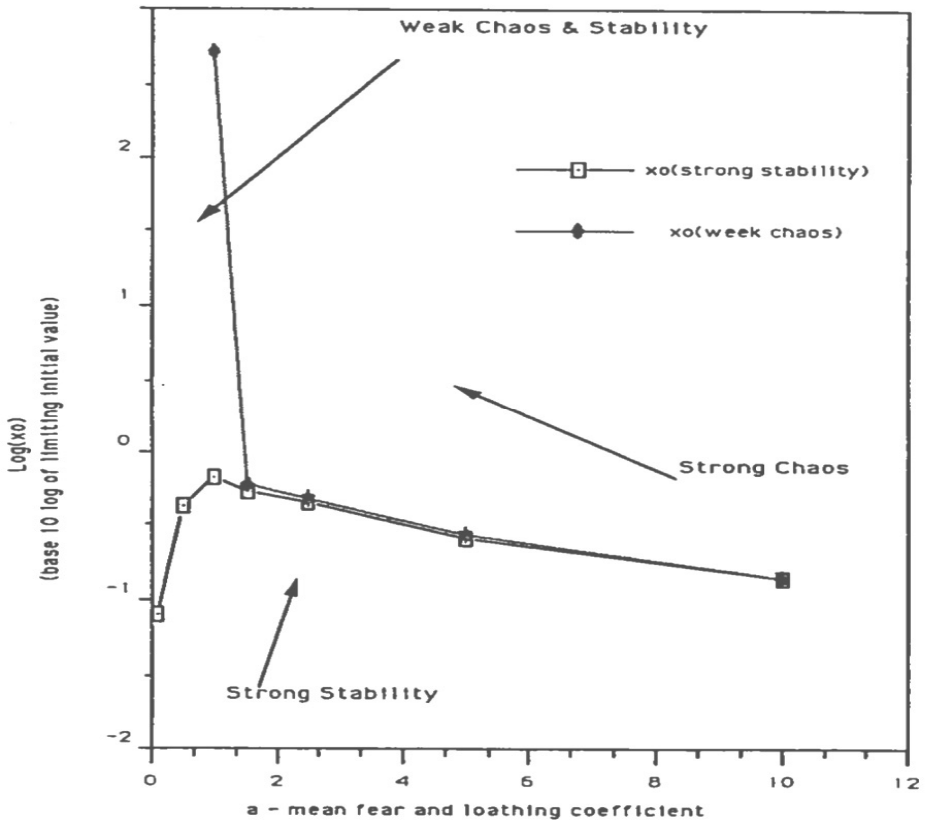

Figure 22. Stability Plot for a Model of Three Independent Competing Nations.[185]

For the second question, "Are democracies more or less prone to war?," we have the same procedure.

$$X_{N+1} = X_N + a_{xy} Y_N (1 - [X_{N+1} - X_N] / C_x)(Y_N - X_N) - (1 / a_{xy}) X_N (X_N - Y_N)$$

$$Y_{N+1} = Y_N + a_{yx} X_N (1 - [Y_{N+1} - Y_N] / C_y)(X_N - Y_N) - (1 / a_{yx}) Y_N (Y_N - X_N)$$

[184] Alvin M Saperstein, *Dynamical Modeling of the Onset of War* (Singapore: World Scientific Publishing, 1999), 102.

[185] Source: Saperstein, "Stability Plot for a Bi-polar Competitive System," 108.

Here, Saperstein defines differently the parameters of the equations; he names them *fear and loathing* coefficients, and the equations are more complicated. He uses a method to define the democracy: "the larger the fraction of population which has significant input into matters of peace and war, the more democratic the nation will be considered to be."[186] With the same procedure after the numerical computations we have the result: "an outbreak of war is more likely in a collection of autocratic states than in a similar collection of democratic states."[187] In other words, democracy is the parameter that increases proportionally the international system's stability.

D. COMPARISON OF THE MODEL WITH THE CONVENTIONAL THEORIES

With Saperstein's model we have two results that are paired with different conventional theories. The theory of realism, as mentioned in Chapter IV, supports that the multipolar state system (that is, constructed from more than three poles of power) is inevitable in this world, since it is totally against the basic need of the powerful states to predominate over the weak.[188] With this argument, the answer to the question: "Which is more war prone—a bipolar or a tripolar world?" is the tripolar world. Saperstein agrees with this, and with a different approach (numbers, equations, and plots) he arrives at the same conclusion.

The same mathematical approach from Saperstein for the question "Are democracies more or less prone to war?" agrees with theory of Liberalism. According to liberalists, democracies never fight each other, and cooperation between nations and states is of utmost importance. The model agrees with this theory, and the inputs are measurable quantities that can be calculated easily.

While the model of Saperstein uses equations and numbers, it comes to the same conclusions as the two major conventional theories of the IR field. The theories of

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[186] Ibid.

[187] Saperstein, *Dynamical Modeling of the Onset of War*, 103.

[188] Chantal Mouffe, "*Democracy in a Multipolar World*," *Millennium: Journal of International Studies*, vol. 37, no. 3 (2009): 549–561.

realism and liberalism agree with the model that the bipolar world is more stable, and that the democracies are safer, respectively. The same technique gave two different directions to the reader, and that proves that the model gives unbiased outcomes. The inputs are predefined parameters that are indifferent to the outcome, and this can be an advantage for the IR analysis.

One characteristic of human nature is that our feelings may affect our opinion, and our opinion in turn may reduce our analytical ability. As Saperstein proves, the models cannot be affected by feelings, opinions, or ideas, and the only element that affects the outcomes of a model is the inputs that we provide. Chaos Theory in this case is the tool for such a prediction, as the system is complex and contains nonlinear phenomena. The model promises that we may have an objective tool in the future, inspired from a theory of physics, for the IR field.

VI. CONCLUSIONS AND RECOMMENDATIONS

The good news of this research is that Chaos Theory works for political science, and there are elements that IR theorists can copy from physicists to benefit their field. The way that physicists work with Chaos shows that they achieve control of Chaos by increasing their knowledge of the behavior of the chaotic attractors. The better feedback they have, the more accurate control they achieve. For an analyst of international relations that can be the next step, as the better the information we gather, the more accurate our analysis will be.

Saperstein's model has the ability to address specific puzzles in the realm of international affairs that are related with the war onset. In other words, the model is designed to produce diagrams that predict instability as a result of specific actions. The IR analysts cannot use the current model to describe other sorts of IR puzzles such as cooperation under anarchy, conflict resolution, civil war dynamics or domestic politics, however. Solving such puzzles with CT models is not impossible, but the models will need more complex equations to address the more numerous parameters these puzzles present. There are also domains, such as the areas of nationalism and the civil-military relations, in which CT will be of little use. The reason is that the parameters of the equations for such issues are much more complex to be defined and the outcomes will be ambiguous.

Saperstein's model is a complementary tool that aims to help IR analysis. The advantage of conventional IR theories is that they are tested in the real world; however, they can be biased or time consuming. For a mathematical model there is an immediate outcome for specific inputs; this result depends only on the parameters that we have already set. The problem with the models is that they are not the real world. For that reason we can test the IR models for several historic events in order to verify whether the models are able to predict the history. This will give us the opportunity to improve our knowledge for the parameters that are significant for a model and to come closer to the real world.

Physicists have already defined exactly what a chaotic phenomenon is and which systems have chaotic behavior. They use phase-space diagrams, chaotic attractors, fractals, Poincare maps, bifurcation diagrams, and many other tools to track, target, and finally control Chaos. The results have been expanded to other natural sciences like chemistry, electronics, biology, computer science, and this implementation is also satisfying.

For the IR domain, Chaos Theory has satisfying and promising results. The use of the theory can be through models like Saperstein's and the analyst has to imitate his approach. The chaotic phenomena that exist in physics have a strong correlation with political events; however, CT for the IR domain means only prediction of Chaos, not control of Chaos. The unpredictability is an issue that exists in international relations, and Chaos Theory is able to help quantify the unpredictability of an IR situation.

The results of this thesis prove that the theory of Chaos is a universal theory that works both for physics and IR. We are able to predict and control Chaos in physics, and we have several applications of that. For the IR domain we have predictability; however, the models are not mature enough yet to give us solutions for the control of Chaos. We need to improve the next step and to control the complex IR phenomena that exist in the real world. The physics part of this thesis indicates that the Lyapunov coefficients are of utmost importance for physicists. For that reason Saperstein's equations, with some additional coefficients, will achieve a more realistic approach to the real world, which in turn will help to control chaotic phenomena. The more parameters a model contains, the more realistic it is. Further research on different IR cases will show which parameters are important to be included in the new equations.

LIST OF REFERENCES

Aaron, Richard. *John Locke*. Oxford: Oxford University Press, 1972.

Abraham, Ralph. "The Chaos Revolution: a Personal View." Chaostory Rev 3.1. Based on a lecture at Kyoto University, March 20, 1998.

———. "The Peregrinations of Poincare." Mathematics Department, University of California, Santa Cruz, 2012. Accessed September 7, 2015. http://www.ralphabraham.org/articles/MS%23136.Poincare/ms136.pdf.

Abraham, Ralph, and Yoshisuke Ueda. *Chaos Avant-Garde: Memoirs of the Early Days of Chaos Theory.* Singapore: World Scientific Publishing, 2000.

Andrievskii, Boris R., and Alexander L. Fradkov. "Methods and Applications." *Automation and Remote Control* 64, no. 5, (2003): 673–713.

Axelrod, Robert, and Robert O. Keohane. "Achieving Cooperation under Anarchy: Strategies and Institutions." In *Neorealism and Neoliberalism: The Contemporary Debate,* edited by David A. Baldwin. New York: Columbia University Press, 1993, 226.

Baker, Gregory L., and Jerry P. Gollub. *Chaotic Dynamics: An Introduction.* 2nd ed. New York: Cambridge University Press, 1996.

Batterman, Robert W. "Defining Chaos." *Philosophy of Science* 60, no. 1 (1993): 43–66.

Brown, Chris. *Understanding International Relations*. 3rd ed. New York: Palgrave McMillan, 2005.

Cranston, Maurice. "Liberalism." In *The Encyclopedia of Philosophy*. Edited by Paul Edwards, New York, New York Press, 1967.

Dendrinos, Dimitrios S. "Cities as Spatial Chaotic Attractors." In *Chaos Theory in the Social Sciences Foundations and Applications*, edited by Douglas Kiel and Euel Elliott. Ann Arbor: The University of Michigan Press, 1996.

Ditto, William L., and Kenneth Showalter. "Introduction: Control and Synchronization of Chaos." *AIP Publishing Chaos* 7 (1997): 509–511.

Ditto, William L., Mark L. Spano, and John F. Lindner. "Techniques for the Control of Chaos," *Physica D*, no. 86 (1995): 198–211.

Ditto, William L., S. N. Rauseo, and Mark L. Spano, "Experimental Control of Chaos," *Physical Review Letters* 65, no. 26 (1990): 3211–3215.

Ferreira, Manuel A. M., Filipe J. A. C. Bonito, Manuel. F. P. Coelho, and Isabel C. Pedro. *Chaos Theory in Politics.* London: Springer Editions, 2014.

Freyberg, Annette, Inan E. Harrison, and Patrick James. *Rethinking Realism in International Relations: Between Tradition and Innovation*. Baltimore, MD: The Johns Hopkins University Press, 2009.

Gleick, James. *Chaos Making a New Science*. 20th anniversary edition. New York: Viking Penguin Inc., 1987.

Glenn, James E. *Chaos Theory: The Essentials for Military Applications*. Newport, RI: Naval War College Press, 1996.

Gross, Paul R., and Norman Levitt. *Higher Superstition*. Baltimore, MD: The John Hopkins University Press, 1994.

Hunt, Raymond E. "Stabilizing High-Period Orbits in a Chaotic System: The Diode Resonator," *Physical Review Letters* 67, no. 15 (1991): 1953–1955.

Kamminga, Harmke. "What Is This Thing Called Chaos?" *New Left Review* I*,* no. 181 (May–June 1990): 46–58.

Kirshner, Jonathan: "Realist Political Economy: Traditions Themes and Contemporary Challenges." In *Routledge Handbook of International Political Economy*, edited by Mark Blyth, 36–47. New York: Routledge, 2009.

Kissane, Dylan. "A Chaotic Theory of International Relations?" *Pro Polis,* no. 2 (2007): 85–103.

Korab-Karpowicz, Julian. "*On the History of Political Philosophy: Great Political Thinkers from Thucydides to Locke."* London: Routledge, 2011.

Lingua Franca. *The Sokal Hoax: The Sham that Shook the Academy*. Lincoln: University of Nebraska Press, 2000.

Locke, John. *The Second Treatise of Government in Two Treatises of Government*, edited by Peter Laslett, 283–285. Cambridge: Cambridge University Press, 1960.

May, Robert M. "Simple Mathematical Models with Very Complicated Dynamics." *Nature* 261 (1976): 459–467.

Meier, Heinrich. *The Lesson of Carl Schmitt: Four Chapters on the Distinction between Political Theology and Political Philosophy*. Chicago: University of Chicago, 1998.

Morgenthau, Hans. *Politics among Nations: The Struggle for Power and Peace.* Boston: McGraw-Hill, 1993.

Mouffe, Chantal. “Democracy in a Multipolar World,” *Millennium: Journal of International Studies* 37, no. 3 (2009): 549–561.

Nozick, Robert. *Anarchy, State, and Utopia*. Oxford: Blackwell, 2013.

Ott, Edward, and Mark Spano. “Controlling Chaos,” *AIP Conference Proceedings* 375, no. 92 (1996): 92–103.

Petrov, Valery, Vlmos Gaspar, Jonathan Masere, and Kenneth Showalter. “Controlling Chaos in the Belousov-Zhabotinsky Reaction.” *Nature* 361 (1993): 240–243.

Roy, Rajarshi, T. W. Murphy, Jr., T. D. Maier, Z. Gills, and Raymond E. Hunt. “Dynamical Control of a Chaotic Laser: Experimental Stabilization of a Globally Coupled System.” *Physical Review Letters* 68, no. 9 (1991): 1259–1262.

Russett, Bruce. *Grasping the Democratic Peace: Principles for a Post- Cold War World.* Princeton, NJ: Princeton University Press, 1993.

Saperstein, Alvin M. *Dynamical Modeling of the Onset of War*. Singapore: World Scientific Publishing, 1999.

Shiraev, Eric B. *International Relations.* New York: Oxford University Press, 2014.

Taylor, John R. *Classical Mechanics*. Colorado: University Science Books, 2005.

Thucydides. *History of the Peloponnesian War*. Translated by Rex Warner. New York: Penguin Books, 1954.

Turci, Luiz F., and Elbert E. Macau. “Chaos Control.” In *Recent Progress in Controlling Chaos*, edited by Miguel A. Sanjuan and Celso Grebogi, 371–94. Singapore: World Scientific Publishing, 2010.

Waltz, Kenneth. *Man, the State, and War: A Theoretical Analysis.* New York: Columbia University Press, 1959.

———. *Theory of International Politics*. New York: McGraw-Hill, 1979.

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